1. [4 pts] Use Newton’s Method, \( p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)} \), to obtain the value of \( \sqrt{4} \) to within 3 decimal places by finding the root of \( f(x) = x^3 - 4 \) using an initial guess of \( p_0 = 4 \). Show the details of your calculation of \( p_1 \), \( p_2 \) and \( p_3 \) and then just the values of the subsequent \( p_n \) values.

\[
P_0 = 4 \\
P_1 = p_0 - \frac{f(p_0)}{f'(p_0)} = 4 - \frac{4 - 4^3 - 4}{3 \cdot 4^2} = 4 - \frac{4 - 60}{48} = 2.75 \\
P_2 = 2.75 - \frac{2.75^3 - 4}{3 \cdot 2.75^2} = 2.0096 \\
P_3 = 2.0096 - \frac{2.0096^3 - 4}{3 \cdot 2.0096^2} = 1.6699 \\
P_4 = \frac{1.5714}{1.5874} \\
P_5 = 1.5874 \\
P_6 = 1.5874 \\
\]

2. Consider a new method of finding a root of an equation, called The Lazy Newton or “Fixed Slope” method. This is similar to Newton’s Method except that instead of taking the derivative at EVERY step, one computes the derivative once at the point of the initial guess \( p_0 \) and uses only that derivative in every subsequent iteration.

The general formula for the Lazy Newton method is: \( p_{n+1} = p_n - \frac{f(p_n)}{f'(p_0)} \)

(a) [4 pts] Indicate on the graph of the function \( f(x) = x^3 - 4 \) below what the first few approximations to the root, \( p_1, p_2, p_3 \) will look like, using the Lazy Newton method, given that \( p_0 = 4 \). Make sure you indicate how you computed \( p_1, p_2 \) and \( p_3 \).

\[
P_0 = 4 \\
P_1 = p_0 - \frac{f(p_0)}{f'(p_0)} = 4 - \frac{4 - 4^3 - 4}{4} = 4.8 \\
P_1 = 2.75 \\
P_2 = 2.75 - \left( \frac{2.75^3 - 4}{4.8} \right) = 2.4001 \\
P_3 = 2.4001 - \left( \frac{2.4001^3 - 4}{4.8} \right) = 2.1954 \\
\]

(b) [2 pts] Use Lazy Newton’s Method to find the value of \( \sqrt{4} \) to within 3 decimal places. Which method do you expect to be faster, Newton’s or Lazy Newton’s? Which one “converges” faster to \( \sqrt{4} \)?

Newton’s takes 4 steps.

Lazy Newton’s take 31 steps to approach \( |p_n - p_{\text{true}}| < 5 \times 10^{-4} \)

\[
P_{31} = 1.590 \\
\]

NOTE: \( 1.590^2 - 4 = 0.018 \)

\( 1.5874^2 - 4 < 0.003 \)

Newton’s is faster, unsurprisingly