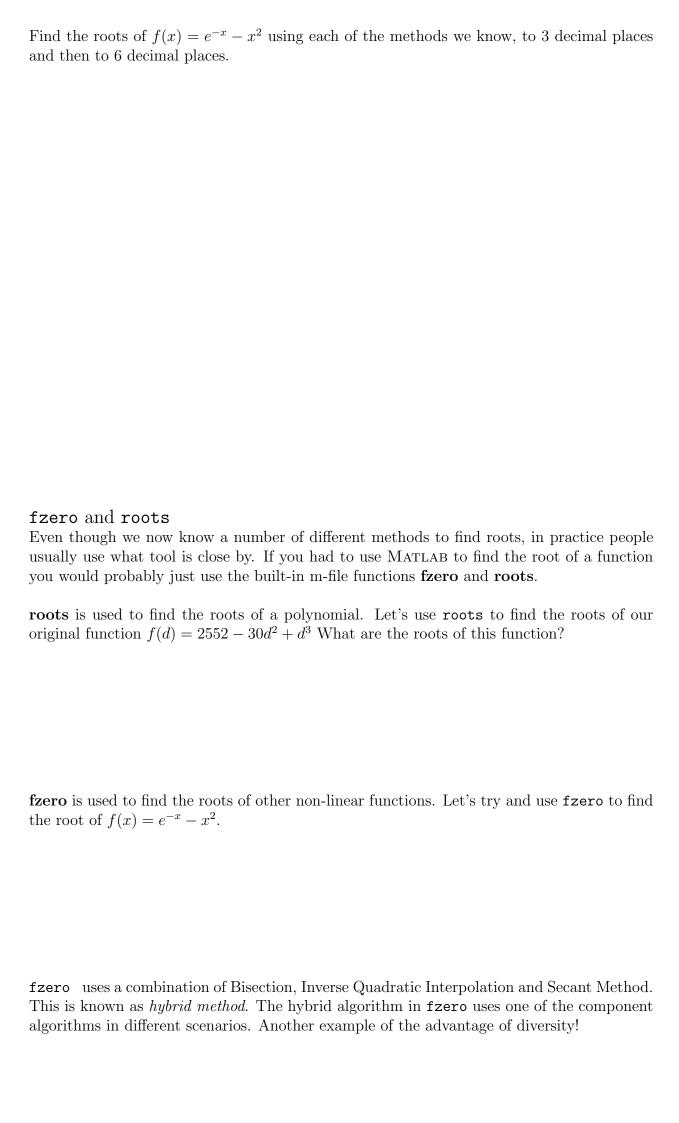
$Numerical\ Analysis$

Math 370 Fall 2004 ©2004 Ron Buckmire $\begin{array}{c} \mathrm{MWF}\ 2:30\ \text{-}\ 3:25\mathrm{pm} \\ \mathrm{Fowler}\ \mathrm{North}\ 5 \end{array}$

Worksheet 12

SUMMARY Comparing Root-finding methods READING Recktenwald, 6.1.1 (240-250)
We have considered iterative methods in this class so far Write them down below with their corresponding iterative step $p_n = g(p_{n-1})$
Let's rank these in order of how fast they converge to the root, i.e. in asymptotic order.
Question
Question How can we prove this order? Can we determine the order of these methods analytically can we do it experimentally? How???
This technique is called



Recall

If we have a sequence of approximations $\{p_n\}$ which converges to p and there exist positive constants α and λ so that

$$\lim_{n \to \infty} \frac{|p_{n+1} - p|}{|p_n - p|^{\alpha}} = \lambda$$

then the sequence is said to **converge to p with order** α , with an **asymptotic error constant** λ . If we define a related sequence $e_n = p_n - p$ representing how far from the "answer" we are, or the error involved, then we can think about this definition in another way.

Summary

In other words the iterative method is said to be of order α if one can show a relationship like $|e_{n+1}| \approx \lambda |e_n|^{\alpha}$

Bisection Method

Derive the error formula for the bisection method and write it below. In other words, get an expression for e_{n+1} in terms of e_n and or n.

Therefore, Bisection is a _____ method, with $\lambda =$ ____ and $\alpha =$ ____

Fixed Point Iteration

We shall derive the asymptotic rate of convergence for Functional Iteration.

$$p_{n+1} = g(p_n)$$
 and $e_{n+1} = p - p_{n+1}$ and $e_n = p - p_n$ therefore $p_{n+1} = g(p - e_n) =$

Newton's Method

In a similar fashion, we shall derive the asymptotic rate of convergence for Newton's Method and fill-in the table below

Method	Order of	Error
	Convergence	Formula
Bisection		
False Position	1.442	
	_	
Secant	$\frac{1+\sqrt{5}}{2} \approx 1.618$	
Newton's		
Picard		

NOTE: The above only apply for simple roots (i.e. a root of multiplicity 1).

Definition

A root r of an equation f(r)=0 has multiplicity m if and only if $0=f(r)=f'(r)=\cdots=f^{(m-1)}(r)=0$ but $f^{(m)}(r)\neq 0$.

For roots of multiplicity m > 1 Newton's Method has the relationship that $|e_{n+1}| \approx \frac{M-1}{M} |e_n|$