Numerical Analysis

Math 370 Fall 2004 MWF 2:30 - 3:25pm
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Worksheet 12

SUMMARY Comparing Root-finding methods
READING Recktenwald, 6.1.1 (240-250)

We have considered ___ iterative methods in this class so far...
Write them down below with their corresponding iterative step $p_n = g(p_{n-1})$

Let's rank these in order of how fast they converge to the root, i.e. in asymptotic order.

Question
How can we prove this order? Can we determine the order of these methods analytically? can we do it experimentally? How???

This technique is called ______________________.

Find the roots of $f(x) = e^{-x} - x^2$ using each of the methods we know, to 3 decimal places and then to 6 decimal places.

`fzero` and `roots`

Even though we now know a number of different methods to find roots, in practice people usually use what tool is close by. If you had to use MATLAB to find the root of a function you would probably just use the built-in m-file functions **fzero** and **roots**.

roots is used to find the roots of a polynomial. Let's use `roots` to find the roots of our original function $f(d) = 2552 - 30d^2 + d^3$ What are the roots of this function?

fzero is used to find the roots of other non-linear functions. Let's try and use `fzero` to find the root of $f(x) = e^{-x} - x^2$.

`fzero` uses a combination of Bisection, Inverse Quadratic Interpolation and Secant Method. This is known as *hybrid method*. The hybrid algorithm in `fzero` uses one of the component algorithms in different scenarios. Another example of the advantage of diversity!

Recall
If we have a sequence of approximations $\{p_n\}$ which converges to p and there exist positive constants α and λ so that

$$\lim_{n\to\infty} \frac{|p_{n+1} - p|}{|p_n - p|^\alpha} = \lambda$$

then the sequence is said to **converge to p with order** α, with an **asymptotic error constant** λ. If we define a related sequence $e_n = p_n - p$ representing how far from the "answer" we are, or the error involved, then we can think about this definition in another way.

Summary
In other words the iterative method is said to be of order α if one can show a relationship like $|e_{n+1}| \approx \lambda |e_n|^\alpha$

Bisection Method
Derive the error formula for the bisection method and write it below. In other words, get an expression for e_{n+1} in terms of e_n and or n.

Therefore, Bisection is a ____________ method, with $\lambda =$ ______ and $\alpha =$ ______

Fixed Point Iteration
We shall derive the asymptotic rate of convergence for Functional Iteration.
$p_{n+1} = g(p_n)$ and $e_{n+1} = p - p_{n+1}$ and $e_n = p - p_n$ therefore
$p_{n+1} = g(p - e_n) =$

Picard Iteration is a ____________ method with $\alpha =$ __________________

Newton's Method

In a similar fashion, we shall derive the asymptotic rate of convergence for Newton's Method and fill-in the table below

Method	Order of Convergence	Error Formula
Bisection		
False Position	1.442	
Secant	$\frac{1+\sqrt{5}}{2} \approx 1.618$	
Newton's		
Picard		

NOTE: The above only apply for simple roots (i.e. a root of multiplicity 1).

Definition

A root r of an equation $f(r) = 0$ has multiplicity m if and only if
$0 = f(r) = f'(r) = \cdots = f^{(m-1)}(r) = 0$ but $f^{(m)}(r) \neq 0$.

For roots of multiplicity $m > 1$ Newton's Method has the relationship that $|e_{n+1}| \approx \dfrac{M-1}{M}|e_n|$