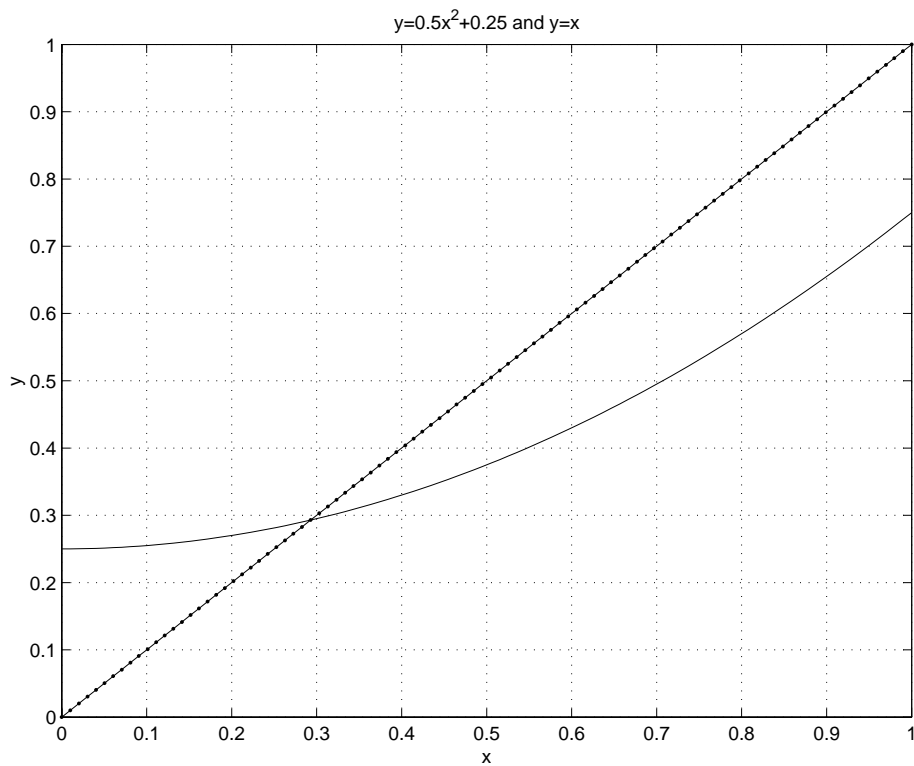
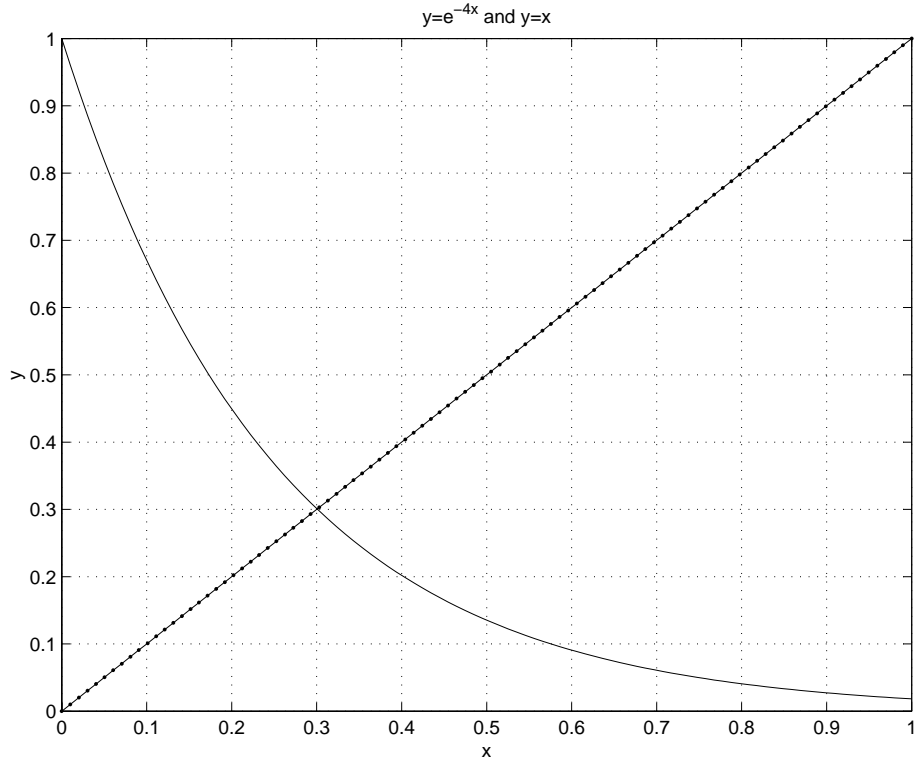
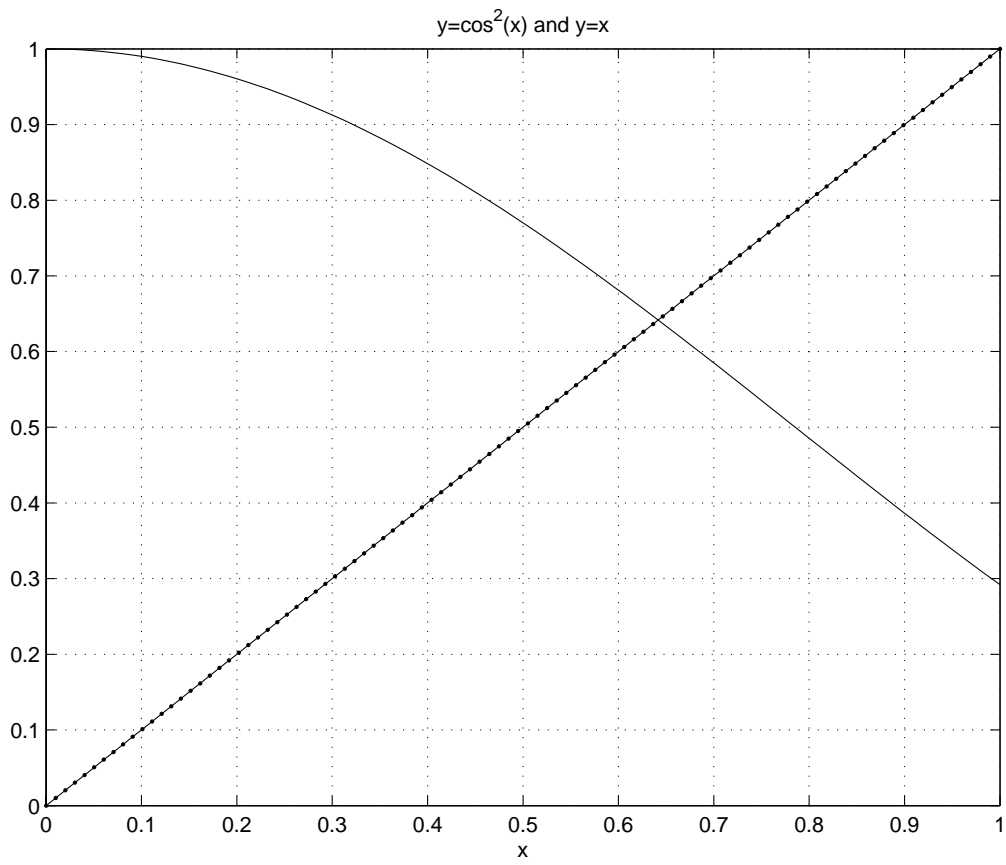
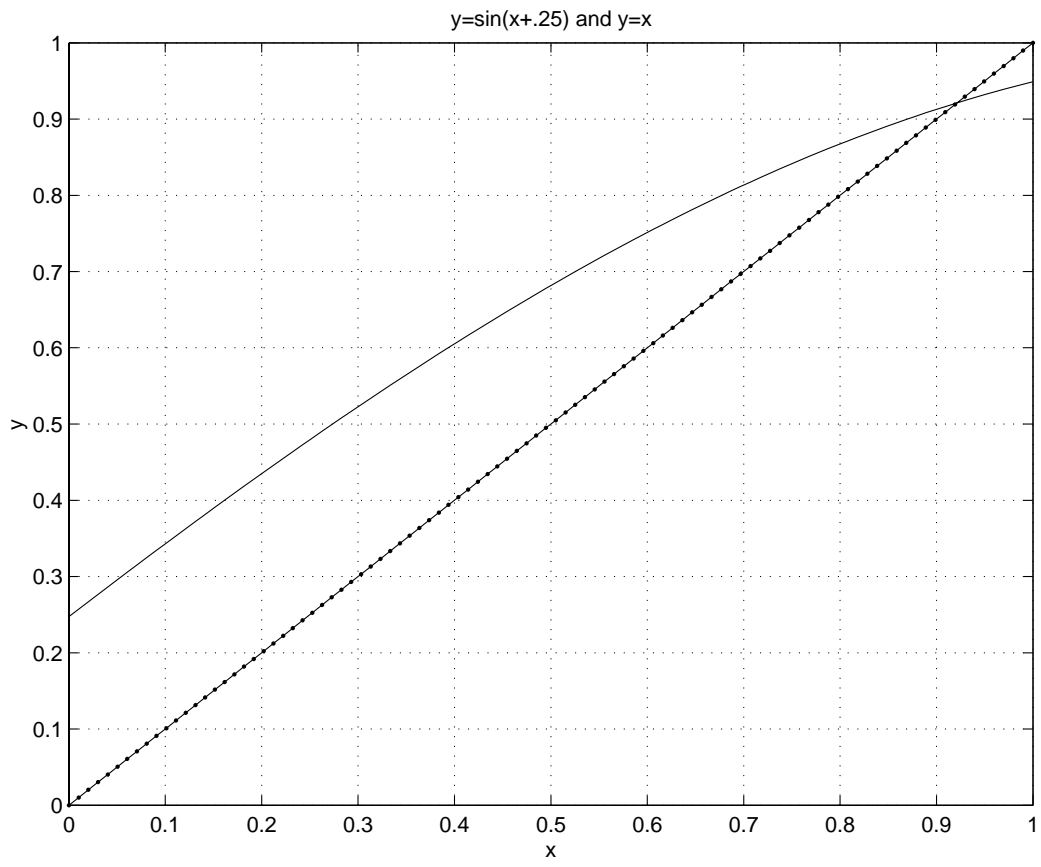


GROUPWORK

Take a look at the following examples of possible functions to do fixed-point iteration on and in groups of two or three graphically indicate what happens. Try the initial guesses like $p_0 = 0$ and $p_0 = 1$.





Convergence Criteria for Picard Iteration

The iteration process $p_n = g(p_{n-1})$ for $n = 1, 2, 3, \dots$ will converge to a unique solution for any initial value p_0 in $[a, b]$ if g' exists on (a, b) and $0 < |g'(x)| < 1$ for all $x \in [a, b]$

Using the above theorem what can we say about our example iteration process?

The example functions were

$$\begin{array}{ll} g_1(x) = \frac{1}{2}x^2 + \frac{1}{4} & g'_1(x) = \\ g_2(x) = e^{-4x} & g'_2(x) = \\ g_3(x) = \cos^2(x) & g'_3(x) = \\ g_4(x) = \sin\left(x + \frac{1}{4}\right) & g'_4(x) = \end{array}$$

Which of these functions converged using functional iteration?

Can you explain why?

(Think about the behavior of the derivative on the interval of interest in each case)

Which of the iterations exhibit **monotone convergence**?

Which of the iterations exhibit **oscillating convergence**?

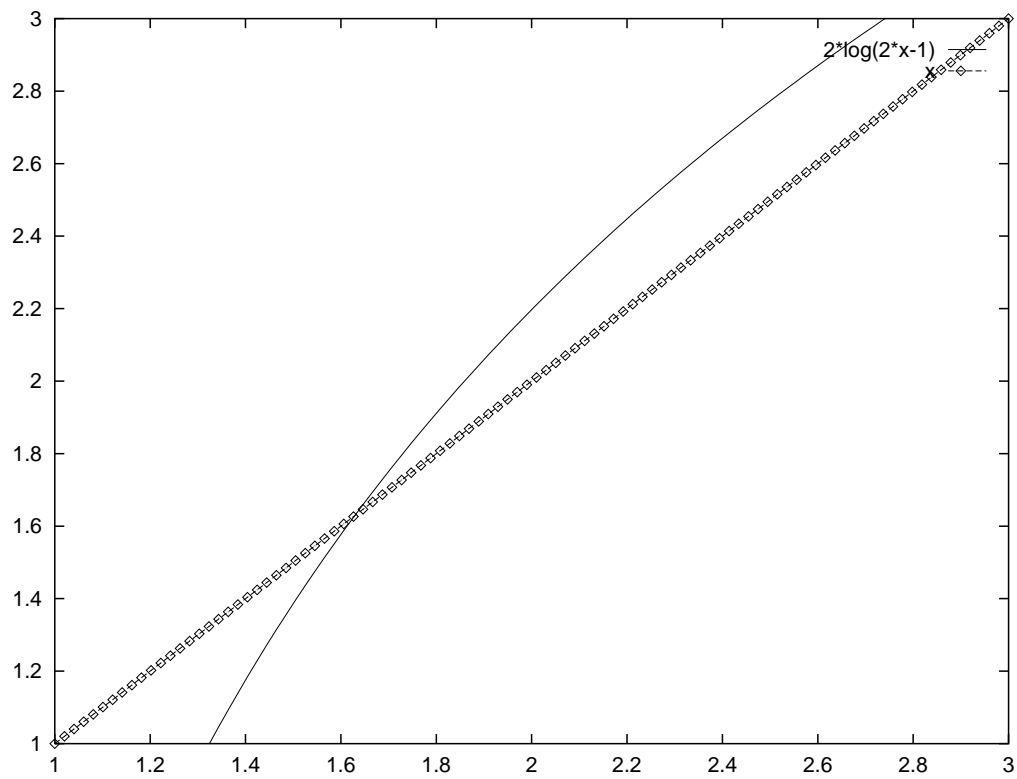
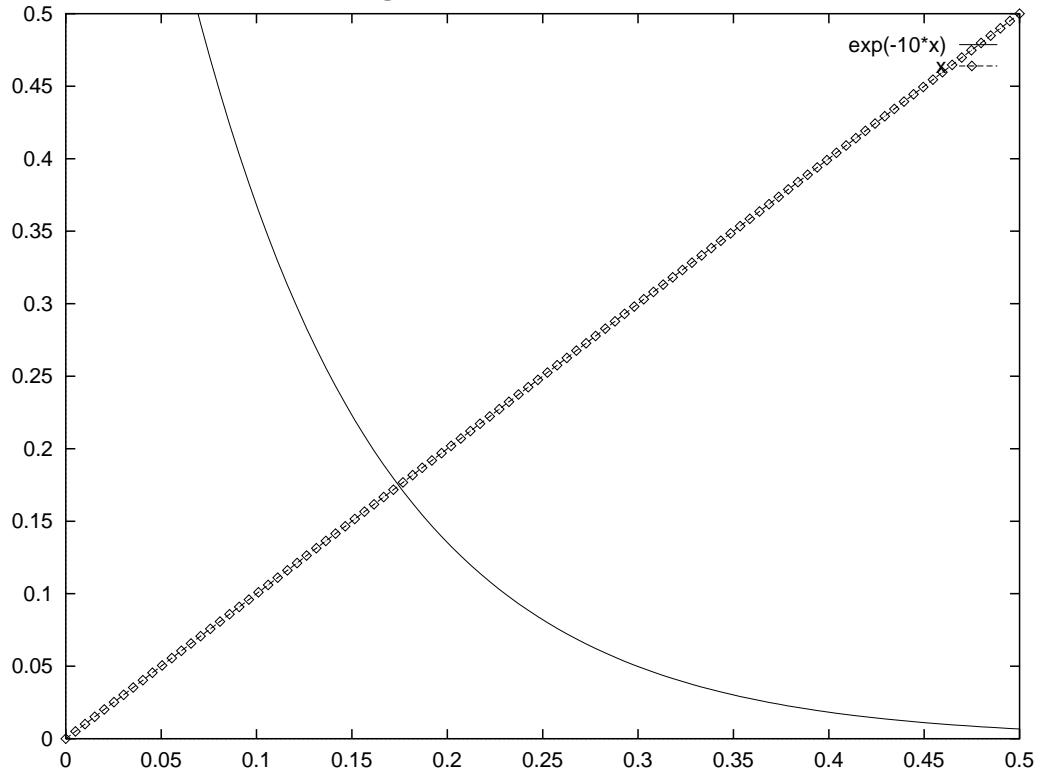
Consider these two other functions,

$$g_5(x) = 2 \ln(2x - 1) \text{ on } [0, 0.5] \text{ and } g_6(x) = e^{-10x} \text{ on } [1, 3]$$

Will Picard Iteration converge or diverge for these examples?
(prove your answer graphically on the next page)

Graphical Examples

Look at these plots of $g_5(x) = 2\ln(2x - 1)$ and $g_6(x) = e^{-10x}$ and graphically indicate whether Picard iteration converges or diverges. In either case classify the convergence as either **monotone** or **oscillating**



EXAMPLE

1. Let us try to use Picard Iteration to approximate $\sqrt[5]{7}$, assuming $p_0 = 1$
2. What function $f(x)$ would we have to use to find a zero for in order to compute $7^{1/5}$?
3. What function would we have to use to do functional iteration on? [i.e. what is $g(x)$?]
Is there only one such function? If you can, write down three possible $g(x)$ functions which have the fixed point $7^{1/5}$
4. How will you decide which function to do the functional iteration on? [i.e. which one will converge the fastest]
5. Try running `picard.m` on your choices and see if this confirms your choice of function above...