Numerical Analysis

Math 370 Fall 2004 MWF 2:30 - 3:25pm

 Fowler North 5

Report on Exam 2

Point Distribution (N=14)

Range	93+	90+	87+	83+	80+	77+	73+	70+	70-
Grade	A	A-	B+	B	B-	C+	C	C-	D
Frequency	5	4	0	1	4	0	0	0	0

Comments

Overall The average score on the exam was a 90 with a standard deviation of 7. The highest score was a 100 (two people!) and the low score was 80. Needless to say, these are pretty astonishing results for a "Buckmire exam!"

#1 Clearly there was only one main problem on the exam: solving nonlinear systems of equations. It was split up into three areas: **theory, application and analysis.** The first theory-based question involved you demonstrating your understanding and facility with the notation involved in depicting how Newton's Method for Systems works and thus being able to modify it to the (simpler) problem of denoting Lazy Newton ("Fixed Jacobian"). There are a number of these modified Newton's, also known as quasi-Newton Methods that are used. The main point to realize on this problem is that the Jacobian is only evaluated at $\vec{x}^{(0)}$ so that any partial derivative present in the formula must be evaluated at the same spot. Also, parenthesesn around the supercripts really are necessary, or else your notation could be interpreted as exponentiation instead. Part (b) is just a slightly more concrete problem of writing down how to compute a SPECIFIC, numbered iterate ($\vec{x}^{(3)}$) from the PREVIOUS one ($\vec{x}^{(2)}$).

#2 The second question gives you a nonlinear system to apply your quasi-Newton's Method to. I assist you by deliberately asking you to compute quantities that you need to know: $f(\vec{\vec{x}}^{(0)})$ and $J(\vec{\vec{x}}^{(0)})$, as well as two you *should* know how to compute but aren't really necessary to solve the problem, i.e. the norms of these last two vectors. Interestingly, the oh-so-detailed formula that you wrote down in question 1 was not used by very many of you to actually compute values of the first two iterates, $\vec{x}^{(1)}$ and $\vec{x}^{(2)}$. Since the initial Jacobian is diagonal, most of you found it easier to invert it and notice the interesting fact that $J = J^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.

#3 The third question continues your work with the nonlinear system by asking you to analyze your results. You are given the analogous results of using regular Newton's Method to solve the nonlinear system. In part (a) you are asked to show that $\vec{x}_{exact} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is the solution, which you can do by evaluating $\vec{f}(\begin{bmatrix} 1 \\ 1 \end{bmatrix})$ and show that it equals $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$. In part (b), you are asked to look at the error being made between the iterates produced using Lazy Newton's Method. This means you are being asked to find the distance between the exact solution $\vec{x}_{exact}$ and $\vec{x}^{(1)}$ and $\vec{x}^{(2)}$, in other words, $||\vec{x}_{exact} - \vec{x}^{(1)}||$ and $||\vec{x}_{exact} - \vec{x}^{(2)}||$. Finding $||\vec{x}_{exact} - \vec{x}^{(0)}||$ wouldn't have hurt either. By computing these values, and or by looking at where they appear on the graph on page 6 compared to the location of the exact solution (which occurs where the graphs intersect, of course) one is able to see that Lazy Newton is producing approximations which are getting no closer to the solution point while Newton produces approximations which gets to the EXACT SOLUTION in two steps. This shouldn't be too surprising, since Newton's uses more information about the problem on each iteration than Lazy Newton's one would expect it to converge faster.