Numerical Analysis

Math 370 Fall 2002 — MWF 9:30am - 10:25pm

 — Fowler 127

Report on Exam 1

Point Distribution (N=14)

Range	90+	80+	75+	65+	60+	55+	45+	40+	40-
Grade	A	A-	B+	B	B-	C+	C	C-	D
Frequency	1	4	1	3	0	1	2	1	1

Comments

Overall The average score on the exam was a 69 with a standard deviation of 19. The highest score was a 97.

#1 This problem tried to connect the concept of "big oh" to limits. This time the function involved was $F(h) = e^{e^h - 1} = 1 + \mathcal{O}(h)$. By telling you how fast $F(h)$ goes to its limit 1 as $h \to 0$ will allow you to immediately evaluate related limits. For example, **(a)** Must evaluate to zero, because basically you're being asked the limit of $F(h) - 1$ which you know looks like $\mathcal{O}(h)$ so it also goes to 0 as $h \to 0$. **(b)** Here you have a race between $h \to 0$ and $F(h) - 1 = \mathcal{O}(h)$ going to zero. From the definition of "big oh" you know this answer must be a finite constant. To narrow down your answer to a number you can use L'Hôpital's Rule or obtain a Maclaurin Series expansion for $e^{e^h - 1} = e^{1 + h + \frac{h^2}{2} + \ldots - 1} = e^{h + \frac{h^2}{2} + \cdots} = e^h e^{h^2/2} e^{h^3/6} \ldots = (1 + h + \dfrac{h^2}{2} + \ldots)(1 + \dfrac{h^2}{2} + \ldots)(1 + \dfrac{h^3}{6} + \ldots)\ldots$ and take $h \to 0$ it's very clear you'll just be left with 1 as your answer. **(c)** Since you know that $F(h) - 1$ is going to zero at $\mathcal{O}(h)$ and the denominator is going to zero as h^2 (which goes to its limits 0 faster than h goes to 0) you know the denominator will "win" and thus the final limit will be infinitely large.

#2 This problem is basically a deep question about functional iteration and fixed points. The goal is to try and classify what happens to every initial value x_0 we enter into the iteration, what value x_∞ does it become? Fixed points are points which do not change under functional iteration so you find those first. **(a)** Solve $g(x) = x$ which means solving $2x(1 - 2x) = x$ and $x - 4x^2 = 0$ which is $x(1 - 4x) = 0$ so $x = 0$ and $x = 1/4$are fixed points. So there are two magic values on the number line, thus splitting the remainder into three regions. You need to check the behavior of the functional iteration on these three region to establish their behavior. So you should run a numerical experiment with $x_0 < 0$ (but close to 0), another with $x_0 > 0$ (but close to 0), another with $0 < x_0 < 0.25$ but close to 0.25 and finally a value $x_0 > 0.25$ but close to 0.25. You'll see that numbers in the first experiment end up getting more and more negative (off to negative infinity) while all the other three end up at 0.25. Thus (0,0) is a repulsive fixed point and (0.25,0.25) is an attractive fixed point. **(c)** The graph is given so that you can confirm your numerical experiments in (b) and also visually see the location of the fixed points in (a). It also points out that there is an upper limit on the "domain of attraction" of the fixed point of $x_0 = 0.25$. When $x_0 = 0.5$, $x_1 = g(x_0) = 0$ which means that $x_\infty = 0$. This is the switchover point so that every initial value x_0 greater than $x_0 = 0.5$ will produce a sequence where x_∞ is $-\infty$.

#3 TRUE/FALSE questions are not by definition easy! Whenever you are taking a test you should be executing an optimization algorithm to maximize your points. In TRUE/FALSE questions this means that what you write to support your determination of TRUE or FALSE needs to be correct and indicate how much you know about the material. For a statement to be TRUE it must always be TRUE, for a statement to be FALSE allyou need to do is think of one example which makes the statement false. **(a)**. It is true that Newton's Method is quadratically convergent, compared to the linear convergence of Bisection. But this is only true when both algorithms are running on equivalent functions for which they both have an equal opportunity to converge. Bisection, once given a bracket, MUST eventually converge. There is no such guarantee with Newton's Method. If you chose an initial x_0 such that $f'(x_0) = 0$ then Newton's Method would fail instantly. Therefore, the given statement is **FALSE.** **(b)** While we were initially experimenting with MATLAB in class, Karl McMurtry noticed that it will represent some numbers smaller than `realmin` without producing underflow. These numbers are known as *denormals* and Professor Recktenwald sent me an email that I posted to Blackboard and sent to the class mailing list. They are discussed in page 196-197 of the text. For example, `realmin/100` will return a value in MATLAB. Therefore, the given statement is **TRUE**. **(c)**. On Worksheet #8 we looked at $p_n = 1/n^2$ and $q_n = 1/2^n$ and showed that both sequences linearly converge to zero. However, clearly q_n will take less steps to reach any given tolerance than p_n will. So the given statement is **FALSE**.