Test 2: Numerical Analysis

Math 370 Monday November 20, 2000

Name: ____________________

Directions: Read *all* 3 (three) problems first before answering any of them. This is a one hour, open-notes, open book, test. You have 90 minutes to complete it. You must show all relevant work to support your answers. Use complete English sentences and indicate your final answer from your "scratch work."

No.	Score	Maximum
1		20
2		50
3		30
Total		100

1. *[20 points total.]* **Interpolation.**

The following data is used in Question 1 and 2.

i	x_i	y_i
0	-1	1/2
1	0	1
2	1	2

Besides Lagrange Interpolating Polynomials there are many other types of basis functions which can be used to form interpolating polynomials. Consider the Newton Interpolating Polynomial, $N(x)$. It has the form

$$N(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2) + \dots$$

(a) *[3 pts]* For the given data, write down the general Newton interpolating Polynomial of degree 2.

(b) *[15 pts]* For the given data, find the second degree Newton interpolating Polynomial $N(x)$.

(c) *[2 pts]* Recall, the form of the Lagrange Interpolation Polynomial (from Quiz 9) was $L(x) = y_0 L_{2,0}(x) + y_1 L_{2,1}(x) + y_2 L_{2,2}(x) = 1 + \frac{3}{4}x + \frac{1}{4}x^2$. How is $L(x)$ related to $N(x)$?

2. *[40 pts. total]* **Approximation Theory.**

This problem involves choosing the curve of best fit for the given data from **(1)**. The choices are between $y = ax + b$ and $y = \dfrac{1}{cx + d}$.

(a) *[10 pts]* Compute the value of a which minimizes the least square error between $y = ax + b$ and the given data.

(HINT: Do not use any decimal approximations of fractions or logarithms.)

(b) *[10 pts]* Compute the value of b which minimizes the least square error between $y = ax + b$ and the given data.

(c) *[4 pts]* Convert the form of the equation from a non-linear form to a linear form; that is from $y = \dfrac{1}{cx+d}$ to $Y = AX + B$. Write down relationships between y, x, c, d and Y, X, A, B, respectively.

(d) *[10 pts]* Compute the value of A which minimizes the least square error between $Y = AX + B$ and the given data.

(b) *[10 pts]* Compute the value of B which minimizes the least square error between $Y = AX + B$ and the given data.

(e) *[6 pts]* Which curve of best fit would you choose to approximate the data? Why?

3. *[30 pts. total]* **Systems of Equations.**

Use Newton's Method with an initial guess of $\vec{x}_0 = (0,0,0)^T$ and find two approximations, $\vec{x}_1$ and $\vec{x}_2$ to the exact solution to $\vec{f}(\vec{x}) = 0$, where

$$\vec{f}(\vec{x}) = \begin{pmatrix} f_1(x,y,z) \\ \\ f_2(x,y,z) \\ \\ f_3(x,y,z) \end{pmatrix} = \begin{pmatrix} -2x + xyz + 1 \\ \\ 2y + xy + yz - 4 \\ \\ -4z + y^2 + 3 \end{pmatrix}.$$

Use Newton's Method with an initial guess of $\vec{x}_0 = (0,0,0)^T$ and find an approximation, $\vec{x}_1$ to the exact solution to $\vec{f}(\vec{x}) = 0$.

(a) *[10 pts]* Compute the jacobian matrix.

(b) *[10 pts]* Use Newton's Method to find your first approximation, $\vec{x}_1$

(d) *[5 pts]* Show that the vector $\vec{r} = (1,1,1)^T$ is the root of $\vec{f}(\vec{x})$

(e) *[5 pts]* Evaluate $||\vec{x}_1 - \vec{r}||_1$, $||\vec{x}_1 - \vec{r}||_2$ and $||\vec{x}_1 - \vec{r}||_\infty$. Do you consider your solution "converged"? What tolerance would you have to use to consider $\vec{x}_1$ a reasonable approximation to $\vec{r}$?