Differential Equations

Math 341 Fall 2014 MWF 3:00-3:55pm Fowler 307
 http://faculty.oxy.edu/ron/math/341/14/

Worksheet 9

TITLE Integrating Factors
CURRENT READING Blanchard, 1.9

Homework Set #5 due Friday September 26 (* denotes Extra Credit Problem)
Section 1.9: 4, 5, 9, 12, 19, 22*.
Chapter 1 Review: 3, 4, 10, 11, 12, 13, 14, 26, 49, 52*.
Section 2.1: 1, 2, 3, 5, 7, 10, 14*.

SUMMARY
We will learn a useful technique for obtaining a formula for solutions of some linear ODEs.

Consider re-writing the standard linear DE $\frac{dy}{dx} = a(x)y + b(x)$ as

$$\frac{dy}{dx} + P(x)y = Q(x) \tag{1}$$

EXAMPLE **Integrating Factor**

It turns out that if one takes the function $\mu(x) = e^{\int P(x)dx}$ and multiplies each term in the modified standard form in (1) by this integrating factor one obtains:

$$\begin{aligned} e^{\int P(x)dx}\frac{dy}{dx} + e^{\int P(x)dx}P(x)y &= e^{\int P(x)dx}Q(x) \\ \frac{d}{dx}\left(e^{\int P(x)dx}y\right) &= Q(x)e^{\int P(x)dx} \\ e^{\int P(x)dx}y &= \int Q(x)e^{\int P(x)dx}\,dx \\ y(x) &= e^{-\int P(x)dx}\int Q(x)e^{\int P(x)dx}\,dx \end{aligned}$$

This is an exact formula for general solutions to the equation in (1).

EXAMPLE

Solve $\frac{dy}{dt} = -2ty + 4e^{-t^2}$

Exercise

Blanchard, page 133, Question 7. Solve $\frac{dy}{dt} = -\frac{y}{1+t} + 2 \quad y(0) = 3.$

GroupWork

Blanchard, page 133, Question 20. For what value(s) of the parameter r is it possible to find explicit formulas (without integrals) for the solution to

$$\frac{dy}{dt} = t^r y + 4?$$