
Differential Equations

Math 341 Fall 2010
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MWF 2:30-3:25pm Fowler 307
<http://faculty.oxy.edu/ron/math/341/10/>

Worksheet 9: Wednesday September 22

TITLE Integrating Factors

CURRENT READING Blanchard, 1.9

Homework Assignments due Friday September 24

Section 1.7: 3, 6, 8, 12, 15.

Section 1.8: 4, 5, 8, 9, 17, 18, 20. Section 1.9: 4, 5, 9, 12, 19.

SUMMARY

We will learn a very cool analytical technique for obtaining a formula for solutions of linear ODEs.

Consider re-writing the standard linear DE $\frac{dy}{dx} = a(x)y + b(x)$ as

$$\frac{dy}{dx} + P(x)y = Q(x) \quad (1)$$

EXAMPLE Integrating Factor

It turns out that if one takes the function $\mu(x) = e^{\int P(x)dx}$ and multiplies each term in the modified standard form in (1) by this integrating factor one obtains:

$$\begin{aligned} e^{\int P(x)dx} \frac{dy}{dx} + e^{\int P(x)dx} P(x)y &= e^{\int P(x)dx} Q(x) \\ \frac{d}{dx} \left(e^{\int P(x)dx} y \right) &= Q(x) e^{\int P(x)dx} \\ e^{\int P(x)dx} y &= \int Q(x) e^{\int P(x)dx} dx \\ y(x) &= e^{-\int P(x)dx} \int Q(x) e^{\int P(x)dx} dx \end{aligned}$$

This is an exact formula for general solutions to the equation in (1).

EXAMPLE

Solve $\frac{dy}{dt} = -2ty + 4e^{-t^2}$

Exercise

Blanchard, page 135, Question 7. Solve $\frac{dy}{dt} = -\frac{y}{1+t} + 2$ $y(0) = 3$.

GROUPWORK

Blanchard, page 135, Question 20. For what value(s) of the parameter r is it possible to find explicit formulas (without integrals) for the solution to

$$\frac{dy}{dt} = t^r y + 4?$$