MATH 341: DIFFERENTIAL EQUATIONS

October 6, 2010

Reading Quiz **TWO**
(Sections 1.5-1.9 of Blanchard)
Prof. R. Buckmire

Your Name: BUCKMIRÉ **SCORE:** /10

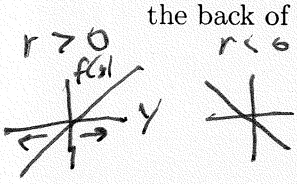
INSTRUCTIONS: Answer the following short-answer questions (in 15 minutes).
GOAL: This reading quiz is designed to illuminate your understanding of the concepts in the second half of the first Chapter of the book: Existence and Uniqueness Theorem, Phase Lines, Equilibria, Bifurcations, Linear Equations, and Integrating Factors.

1. (*2 points.*) In order for a solution $y(t)$ to $y' = f(t,y), \quad y(t_0) = y_0$ to exist AND be unique for all t what conditions must hold? Provide an example of an IVP which has these properties, if you think it is possible.

$y' = y, y(0) = 1$ has solution $y(t) = e^t \ \forall t$

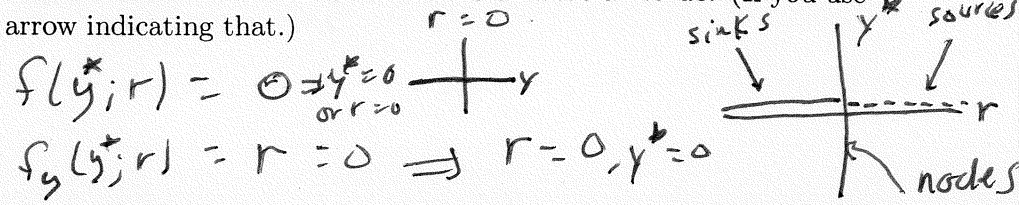
f and f_y continuous everywhere $\Rightarrow$ exist & unique everywhere $f(t,y) = y$ and $f_y(t,y) = 1$ are polynomials thus are continuous everywhere

2. (*2 points.*) Draw the bifurcation diagram for the following ODE $y' = ry$ where r is a parameter. Clearly indicate the stable and unstable sections of the curve and the bifurcation value. (If you use the back of this sheet draw an arrow indicating that.)



$y' = ry = f(y^*; r) = 0 \Rightarrow y^* = 0$ or $r = 0$

$f_y(y^*; r) = r = 0 \Rightarrow r = 0, y^* = 0$



3. (*2 points.*) If it is possible, write down an example of an autonomous, first-order ODE that has no equilibria. EXPLAIN YOUR ANSWER.

$y' = 1$ has no equilibria and is autonomous since $1 \neq 0$

or $y' = e^y$ or $y' = y^2 + 1$ [For what functions is $f(y)$ never zero for any y value? For any values of y]

4. (*2 points.*) TRUE or FALSE: "If $y(t)$ is a solution of a non-homogeneous, linear, ordinary differential equation, then $cy(t)$ (where c is any real number) must also be a solution." PROVE YOUR ANSWER or provide a counter-example.

FALSE! $y' = y + 1 \Rightarrow y(t) = Ae^t + K$ $y' = Ae^t = Ae^t + K + 1$

$0 = K + 1 \Rightarrow K = -1$

$y(t) = Ae^t - 1$

$y' = 2Ae^{2t} \stackrel{?}{=} 2Ae^{2t} - 2 + 1 = 2Ae^{2t} - 1$

LHS $\neq$ RHS

$y(t) = 2Ae^t - 2$ is NOT A SOLUTION TO $y' = y + 1$

5. (*2 points.*) What is the integrating factor $\mu(x)$ that you would have to multiply the ODE $y' = -3y\ln(x) + 1$ by in order to obtain a general solution? [DO NOT SIMPLIFY YOUR ANSWER!]

$\mu = e^{\int 3\ln x}$ is the integrating factor

BONUS (*2 points.*) Comment on the pace of the class. Is it going too fast, too slow or "just right"? What is the concept you think that you least understand in these sections of the text?