Quiz 4 — Differential Equations

Name: ______________________________

Time Begun: ____________________
Time Ended: ____________________

Friday October 22
Prof. Ron Buckmire

Topic : Linear Systems of Equations

The idea behind this quiz is to provide you with an opportunity to illustrate your understanding of solutions of linear systems in 2-D.

Reality Check:

EXPECTED SCORE : __________**/10**   ACTUAL SCORE : __________**/10**

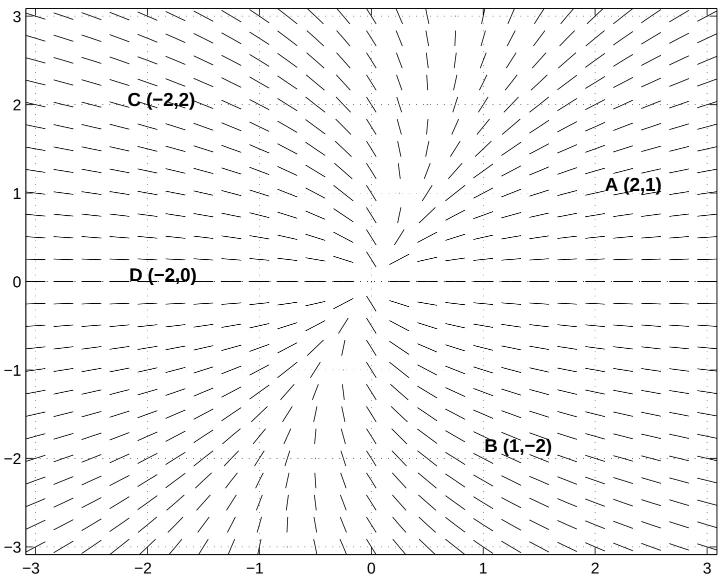
Instructions:

0. Please look for a hint on this quiz posted to `faculty.oxy.edu/ron/math/341/10/`
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday October 25**, in class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. Consider the slope field for the given system

$$\frac{dx}{dt} = -2x + \frac{1}{2}y$$
$$\frac{dy}{dt} = -y$$



(a) *1 point.* Write the given system of linear ODEs in matrix form $\frac{d\vec{x}}{dt} = A\vec{x}$, where $\vec{x} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$.

(b) *4 points.* Find the eigenvalues λ_1 and λ_2 and the eigenvectors $\vec{v_1}$ and $\vec{v_2}$ of the matrix A from part **(a)** and use that information to write the general solution of the system of linear ODEs in the form $\vec{x}(t) = c_1 e^{\lambda_1 t}\vec{v_1} + c_2 e^{\lambda_2 t}\vec{v_2}$

(c) *4 points.* On the given phase plane, indicate the trajectories for solutions which start at the initial conditions $A = (2, 1)$, $B = (1, -2)$, $C = (-2, 2)$ and $D = (-2, 0)$ (**USE ARROWS!**)

(d) *1 point.* Discuss the stability of the equilibrium point (at the origin) of the system. **Is the equilibrium stable or un-stable?** (HINT: What happens to solutions as $t \to \infty$ for each solution curve.)