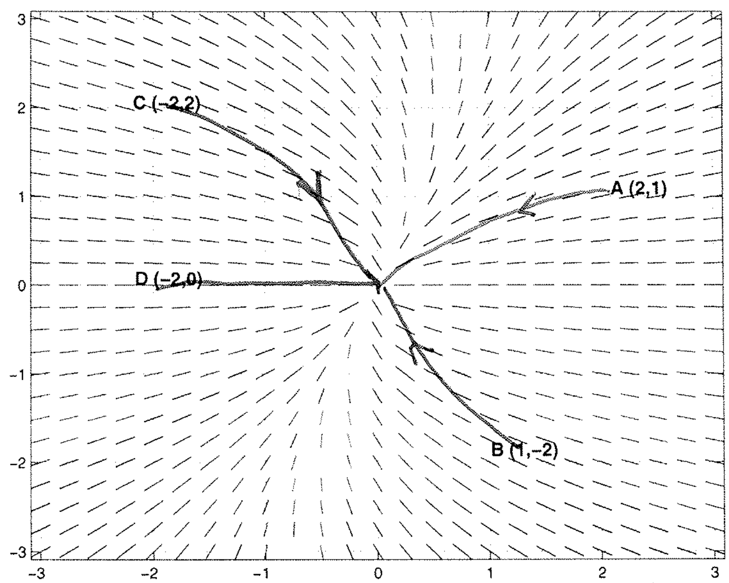
1. Consider the slope field for the given system

$$\frac{dx}{dt} = -2x + \frac{1}{2}y$$
$$\frac{dy}{dt} = -y$$



(a) *1 point.* Write the given system of linear ODEs in matrix form $\frac{d\vec{x}}{dt} = A\vec{x}$, where $\vec{x} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$.

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} -2 & 1/2 \\ 0 & -1 \end{pmatrix} \vec{x}$$

(b) *4 points.* Find the eigenvalues λ_1 and λ_2 and the eigenvectors $\vec{v_1}$ and $\vec{v_2}$ of the matrix A from part **(a)** and use that information to write the general solution of the system of linear ODEs in the form $\vec{x}(t) = c_1 e^{\lambda_1 t}\vec{v_1} + c_2 e^{\lambda_2 t}\vec{v_2}$

Upper triangular so $\lambda_1 = -2$ or $\lambda_2 = -1$

Find $null(A-(-2)I) \Rightarrow \left(\begin{array}{cc:c} 0 & 1/2 & 0 \\ 0 & -1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc:c} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow$ $y=0$, x any
$\lambda = -2$, $E_{-2} = span\left\{\begin{pmatrix}1\\0\end{pmatrix}\right\}$

$null(A-(-1)I) \Rightarrow \left(\begin{array}{cc:c} -1 & 1/2 & 0 \\ 0 & 0 & 0 \end{array}\right) \rightarrow \left(\begin{array}{cc:c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $2x - y = 0$, $y = 2x$, $E_{-1} = \begin{pmatrix}x\\2x\end{pmatrix} = x\begin{pmatrix}1\\2\end{pmatrix}$, $x \in \mathbb{R}$

$$\vec{x}(t) = c_1 e^{-t}\begin{pmatrix}1\\2\end{pmatrix} + c_2 e^{-2t}\begin{pmatrix}1\\0\end{pmatrix}$$

(c) *4 points.* On the given phase plane, indicate the trajectories for solutions which start at the initial conditions $A = (2, 1)$, $B = (1, -2)$, $C = (-2, 2)$ and $D = (-2, 0)$ (**USE ARROWS!**)

(d) *1 point.* Discuss the stability of the equilibrium point (at the origin) of the system. **Is the equilibrium stable or un-stable?** (HINT: What happens to solutions as $t \to \infty$ for each solution curve.)

As $t \to \infty$, $\vec{x}(t) \to \begin{pmatrix}0\\0\end{pmatrix}$ so the origin is STABLE (i.e. a SINK).