Math 341 Fall 2010 **SHOW ALL YOUR WORK** Quiz **One**

1. Consider the following differential equation

$$\frac{dy}{dx} = \left(\frac{y}{x}\right)^2 + 2\frac{y}{x}.$$

(a) *1 point.* Fully classify this differential equation by **type**, **order** and **linearity**.

ODE, 1^{st} order, nonlinear & non-autonomous

(b) *2 points.* Show that the given differential equation when thought of as $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ can be transformed using the transformation $u = y/x$ (i.e. $y = ux$) into a *separable* equation of the form $x\frac{du}{dx} = F(u) - u$ where $F(t) = t^2 + 2t$. (HINT: note that u is a function of x, so the right-hand side of $y = ux$ is also only a functio of x).

$$\frac{dy}{dx} = \frac{d(ux)}{dx} = \frac{du}{dx}.x + u\frac{dx}{dx} = x\frac{du}{dx} + u$$

$$F(y/x) = (y/x)^2 + 2(y/x) \Rightarrow F(u) = u^2 + 2u$$

$$x\frac{du}{dx} + u = u^2 + 2u$$

$$x\frac{du}{dx} = u^2 + 2u - u = u^2 + u$$

(c) *4 points.* Use the separation of variables technique to show that the *general* solution to the given differential equation has the form $y = \frac{Cx^2}{1 - Cx}$, where C is an unspecified constant.

$$\frac{du}{u^2+u} = \frac{dx}{x} \Rightarrow \int \frac{du}{u(u+1)} = \int\frac{dx}{x} = \ln|x| + C$$

$$\frac{1}{u(u+1)} = \frac{A}{u} + \frac{B}{u+1}$$

$$1 = A(u+1) + B\cdot u$$

$u=0$, $A=1$

$u=-1$, $B=-1$

$$\frac{1}{u(u+1)} = \frac{1}{u} - \frac{1}{u+1}$$

$$\int \frac{1}{u} - \frac{1}{u+1}\,du =$$

$$\ln|u| - \ln|u+1| = \ln|x| + C$$

$$\ln\left|\frac{u}{u+1}\right| = \ln x + C$$

$$\frac{u}{u+1} = e^{\ln x + C}$$

$$= xA \quad (A = e^C)$$

$$\frac{u}{u+1}$$

$$u = Ax(u+1)$$

$$u - Axu = Ax$$

$$u(1-Ax) = Ax$$

$$u = \frac{Ax}{1-Ax}$$

$$\frac{y}{x} = \frac{Ax}{1-Ax}$$

$$y = \frac{Ax^2}{1-Ax}$$

$A \equiv C$ unknown constant

(d) *3 points.* If possible, find each of the *particular* solutions to the differential equation which go through the points $(1,1)$, $(1,0)$ and $(0,1)$ in the xy-plane, respectively. DISCUSS YOUR ANSWERS.

$x=1, y=1$

$$1 = \frac{A}{1-A}$$

$$\Rightarrow 1 - A = A$$

$$A = 1/2$$

$$y = \frac{1/2\, x^2}{1 - 1/2\, x}$$

$$y = \frac{x^2}{2-x}$$ is sol^n through $(1,1)$

$x=1, y=0$

$$0 = \frac{A\cdot 1}{1-A} \Rightarrow A = 0 \Rightarrow y = 0$$ is solution through $(1,0)$

$x=0, y=1$

$$1 = \frac{A\cdot 0}{1 - A\cdot 0}$$

impossible!

There is no solution through the point $(0,1)$

A particular sol^n is obtained by checking the general solution of an initial condition