1. We're interested in finding the function $f(t)$ whose Laplace Transform is

$$F(s) = A(s) - B(s) = \frac{1}{s^2} - \frac{e^{-s}}{s(1-e^{-s})}, \quad s > 0$$

(a) *2 points.* Compute $\mathcal{L}^{-1}\left[\frac{1}{s^2}\right] = a(t)$.

$$\mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{t^{n-1}}{(n-1)!}$$

$$a(t) = \frac{t^1}{1!} = t$$

(b) *2 points.* If one considers $\frac{1}{1-e^{-s}}$ as the sum of a geometric series $\sum_{k=0}^{\infty} ar^k$ with first term $a = 1$ and ratio $r = e^{-s}$ then show that $\frac{e^{-s}}{s(1-e^{-s})}$ can be written as $\sum_{k=1}^{\infty} \frac{e^{-ks}}{s} = \frac{e^{-s}}{s} + \frac{e^{-2s}}{s} + \frac{e^{-3s}}{s} + \ldots$

$$\frac{e^{-s}}{s}\frac{1}{1-e^{-s}} = \frac{e^{-s}}{s}\sum_{k=0}^{\infty}(e^{-s})^k = \frac{1}{s}\sum_{k=0}^{\infty}(e^{-s})^{k+1} = \frac{1}{s}\sum_{k=1}^{\infty} e^{-ks}$$

$$= \sum_{k=1}^{\infty}\frac{e^{-ks}}{s} = \frac{e^{-s}}{s} + \frac{e^{-2s}}{s} + \frac{e^{-3s}}{s} + \ldots$$

(c) *3 points.* Recall that $\mathcal{L}^{-1}\left[e^{-as}F(s)\right] = f(t-a)\mathcal{H}(t-a)$. Using the result given in (b), compute $\mathcal{L}^{-1}\left[\frac{e^{-s}}{s(1-e^{-s})}\right] = b(t)$.

$$\mathcal{L}^{-1}\left[\frac{e^{-s}}{s(1-e^{-s})}\right] = \mathcal{L}^{-1}\left[\frac{e^{-s}}{s}\right] + \mathcal{L}^{-1}\left[\frac{e^{-2s}}{s}\right] + \ldots$$

$$= \mathcal{H}(t-1) + \mathcal{H}(t-2) + \mathcal{H}(t-3) + \ldots = \lfloor t \rfloor = \text{lowest integer function}$$

(d) *3 points.* Give a sketch of $a(t)$, $b(t)$ and $f(t) = a(t) - b(t)$ below for $t > 0$ (Use different pairs of axes for each graph.)