28. Both Gib and Harry will follow the solution curve for their own starting point. A careful proof that distinct solution curves do not touch requires the Uniqueness Theorem (see Section 2.4).

29. Since the vector field does not change with time, Gib will follow the same path as Harry, only one time unit behind. See Section 2.4 for the Uniqueness Theorem that rigorously justifies these statements.

EXERCISES FOR SECTION 2.3

1. To check that $dx/dt = 2x + 2y$, we compute both

$$\frac{dx}{dt} = 2e^t$$

and

$$2x + 2y = 4e^t - 2e^t = 2e^t.$$

To check that $dy/dt = x + 3y$, we compute both

$$\frac{dy}{dt} = -e^t,$$

and

$$x + 3y = 2e^t - 3e^t = -e^t.$$

Both equations are satisfied for all t. Hence $(x(t), y(t))$ is a solution.

2. To check that $dx/dt = 2x + 2y$, we compute both

$$\frac{dx}{dt} = 6e^{2t} + e^t$$

and

$$2x + 2y = 6e^{2t} + 2e^t - 2e^t + 2e^{4t} = 6e^{2t} + 2e^{4t}.$$

Since the results of these two calculations do not agree, the first equation in the system is not satisfied, and $(x(t), y(t))$ is not a solution.

3. To check that $dx/dt = 2x + 2y$, we compute both

$$\frac{dx}{dt} = 2e^t - 4e^{4t}$$

and

$$2x + 2y = 4e^t - 2e^{4t} - 2e^t + 2e^{4t} = 2e^t.$$

Since the results of these two calculations do not agree, the first equation in the system is not satisfied, and $(x(t), y(t))$ is not a solution.

4. To check that $dx/dt = 2x + 2y$, we compute both

$$\frac{dx}{dt} = 4e^t + 4e^{4t}$$

and

$$2x + 2y = 8e^t + 2e^{4t} - 4e^t + 2e^{4t} = 4e^t + 4e^t.$$

To check that $dy/dt = x + 3y$, we compute both

$$\frac{dy}{dt} = -2e^t + 4e^{4t},$$

and

$$x + 3y = 4e^t + e^{4t} - 6e^t + 3e^{4t} = -2e^t + 4e^{4t}.$$

Both equations are satisfied for all t. Hence $(x(t), y(t))$ is a solution.

5. The second equation in the system is $dy/dt = -y$, and from Section 1.1, we know that $y(t)$ must be a function of the form $y_0 e^{-t}$, where y_0 is the initial value.

6. Yes. You can always show that a given function is a solution by verifying the equations directly (as in Exercises 1–4).

To check that $dx/dt = 2x + y$, we compute both

$$\frac{dx}{dt} = 8e^{2t} + e^{-t}$$

and

$$2x + y = 8e^{2t} - 2e^{-t} + 3e^{-t} = 8e^{2t} + e^t.$$

To check that $dy/dt = -y$, we compute both

$$\frac{dy}{dt} = -3e^{-t},$$

and

$$-y = -3e^{-t}.$$

Both equations are satisfied for all t. Hence $(x(t), y(t))$ is a solution.

7. From the second equation, we know that $y(t) = k_1 e^{-t}$ for some constant k_1. Using this observation, the first equation in the system can be rewritten as

$$\frac{dx}{dt} = 2x + k_1 e^{-t}.$$

This equation is a first-order linear equation, and we can derive the general solution using the Extended Linearity Principle from Section 1.8 or integrating factors from Section 1.9.

Using the Extended Linearity Principle, we note that the general solution of the associated homogeneous equation is $x_h(t) = k_2 e^{2t}$.

To find one solution to the nonhomogeneous equation, we guess $x_p(t) = \alpha e^{-t}$. Then

$$\begin{aligned}\frac{dx_p}{dt} - 2x_p &= -\alpha e^{-t} - 2\alpha e^{-t} \\ &= -3\alpha e^{-t}.\end{aligned}$$

Therefore, $x_p(t)$ is a solution if $\alpha = -k_1/3$.

The general solution for $x(t)$ is

$$x(t) = k_2 e^{2t} - \frac{k_1}{3} e^{-t}.$$

8. **(a)** No. Given the general solution

$$\left(k_2 e^{2t} - \frac{k_1}{3} e^{-t}, k_1 e^{-t}\right),$$

the function $y(t) = 3e^{-t}$ implies that $k_1 = 3$. But this choice of k_1 implies that the coefficient of e^{-t} in the formula for $x(t)$ is -1 rather than $+1$.

(b) To determine that $\mathbf{Y}(t)$ is not a solution without reference to the general solution, we check the equation $dx/dt = 2x + y$. We compute both

$$\frac{dx}{dt} = -e^{-t}$$

and

$$2x + y = 2e^{-t} + 3e^{-t}.$$

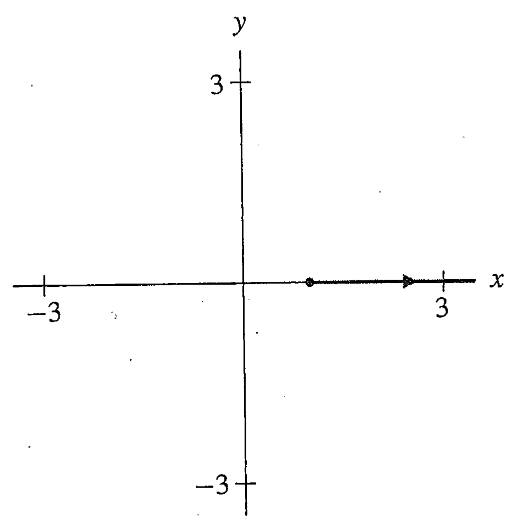
Since these two functions are not equal, $\mathbf{Y}(t)$ is not a solution.

9. **(a)** Given the general solution

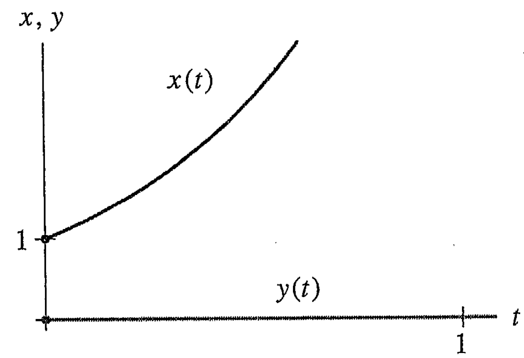
$$\left(k_2 e^{2t} - \frac{k_1}{3} e^{-t}, k_1 e^{-t}\right),$$

we see that $k_1 = 0$, and therefore $k_2 = 1$. We obtain $\mathbf{Y}(t) = (x(t), y(t)) = (e^{2t}, 0)$.

(b)



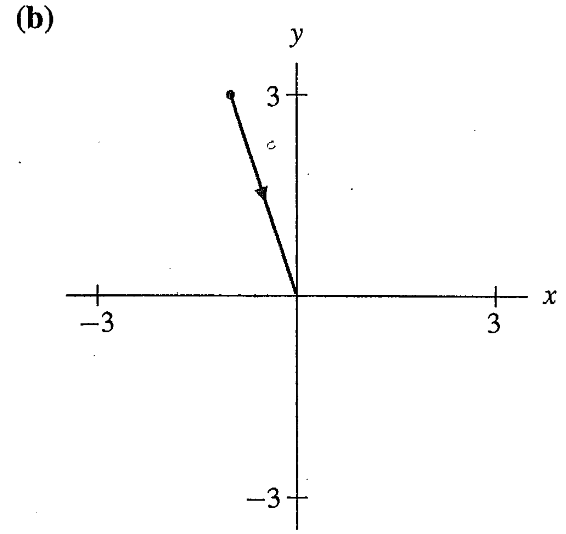
(c)

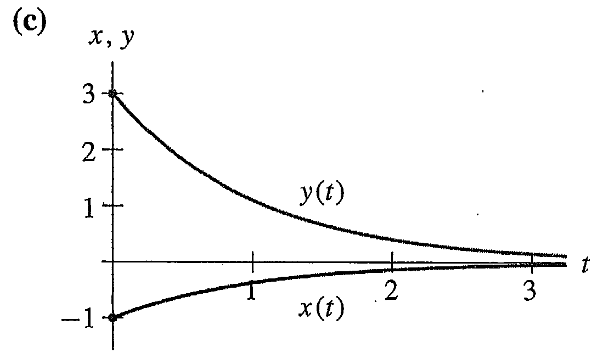


10. **(a)** Given the general solution

$$\left(k_2 e^{2t} - \frac{k_1}{3} e^{-t}, k_1 e^{-t}\right),$$

we see that $k_1 = 3$, and therefore $k_2 = 0$. We obtain $\mathbf{Y}(t) = (x(t), y(t)) = (-e^{-t}, 3e^{-t})$.



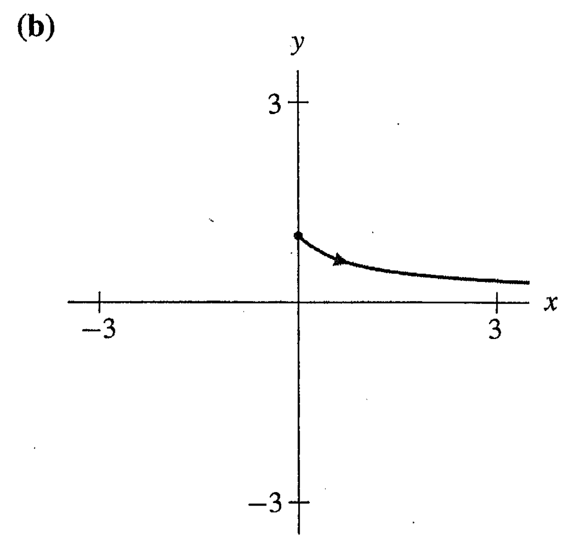


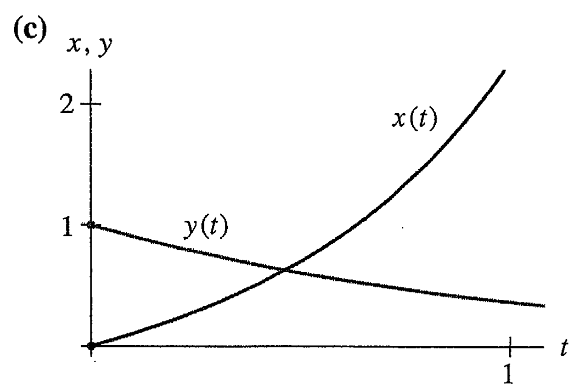
11. **(a)** Given the general solution

$$\left(k_2 e^{2t} - \frac{k_1}{3} e^{-t}, k_1 e^{-t}\right),$$

we see that $k_1 = 1$, and therefore $k_2 = 1/3$. We obtain

$$\mathbf{Y}(t) = (x(t), y(t)) = \left(\tfrac{1}{3} e^{2t} - \tfrac{1}{3} e^{-t}, e^{-t}\right).$$





12. **(a)** Given the general solution

$$\left(k_2 e^{2t} - \frac{k_1}{3} e^{-t}, k_1 e^{-t}\right),$$

we see that $k_1 = -1$, and therefore $k_2 = 2/3$. We obtain

$$\mathbf{Y}(t) = (x(t), y(t)) = \left(\tfrac{2}{3} e^{2t} + \tfrac{1}{3} e^{-t}, -e^{-t}\right).$$