HW #1

Sec 1.1: 3, 4, 8, 11

3. $\frac{dP}{dt} = 0.4P\left(1 - \frac{P}{230}\right)$

(a) Equilibrium occurs when $\frac{dP}{dt} = 0$

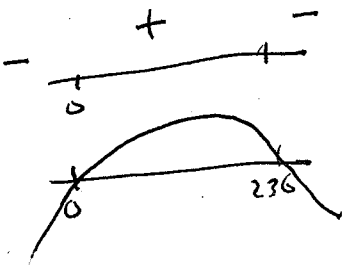
$P = 0$ or $P = 230$

(b) $P\uparrow$ when $\frac{dP}{dt} > 0$

$P\left(1 - \frac{P}{230}\right) > 0$

$P(230 - P) > 0$

$0 < P < 230$



(c) $P\downarrow$ means $P' < 0$

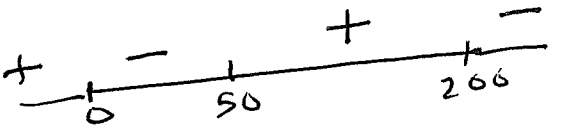
$P\left(1 - \frac{P}{230}\right) < 0$

$P > 230$ or $P < 0$ ← not really possible.

4. $\frac{dP}{dt} = 0.3\left(1 - \frac{P}{200}\right)\left(\frac{P}{50} - 1\right)P$

(a) $P = 0$ or $P = 200$ or $P = 50$ cause $\frac{dP}{dt} = 0$.

(b) $\frac{dP}{dt} < 0$ when $0 < P < 50$ or $P > 200$



(b) $\frac{dP}{dt} > 0$ when $P < 0$ (non-physical)

or

$50 < P < 200$

8. $\frac{dL}{dt} = 2(1-L)$

At $t=0$, $L_1 = \frac{1}{2} \Rightarrow \frac{dL_1}{dt} = 2(1-\frac{1}{2}) = 1$

At $t=0$, $L_2 = 0 \Rightarrow \frac{dL_2}{dt} = 2(1-0) = 2$

$L_2'(0) > L_1'(0)$

So the rate of learning of the second student is faster.

11. $\frac{dr}{dt} = -\lambda r, r(0) = r_0 \Rightarrow r(t) = r_0 e^{-\lambda t}$

(a) For Carbon-14 $r(5230) = \frac{1}{2} r_0$

$$\frac{r_0}{2} = r_0 e^{-\lambda 5230}$$

$$\frac{1}{2} = e^{-\lambda 5230}$$

$$\ln\left(\frac{1}{2}\right) = -\lambda 5230$$

$$-\ln 2 = -\lambda 5230$$

$$\frac{\ln 2}{5230} = \lambda$$

(b) For Iodine-131

$$\lambda = \frac{\ln 2}{8}$$

(c) Units of λ are (time^{-1}) or in (a) per year or in (b) "per day"

(d) It always takes the same time to decay to "half" the size of the initial sample, called the "half-life."