Report on Test 2 Prof. Ron Buckmire

Point Distribution (N=21)

Range	50+	42+	40+	38+	35+	34+	33+	30+	28+	25+	20+	15+	15-
Grade	A+	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Frequency	0	2	1	3	3	3	1	3	0	0	2	1	0

Summary This exam was a very near duplicate of the 2009 Exam #2 on which the median was 33 and the median was 32.6. The results in the 2010 exam were strikingly siimilar, with the median score being 34 and the average exactly 33. The high score on last year's exam was a 45, this year it was a 43. Although it may be rare at Occidental, it is not rare in many places to have exam score distributions which resemble those on this exam. As I said in class on Day 1, "my exams are hard" because they involve students to use the knowledge and skills that they have just developed in the class and apply them in what appears to be an unfamiliar context. The ability to do this involves creativity and critical thinking and remaining calm in the face of adversity. If you stick to applying everything you know is always true, you should be able to deal with "Buckmire-type" exams. I understand and appreciate that the experience while taking the exam is an unpleasant one, but I do strongly believe being able to reduce one's knowledge about a particular course into a key set of ideas that one can apply in multiple settings, no matter how bizarre they seem at first is really more important than the specific content material before you. To balance the effect of my exams on the course grade, I make exams a small percentage of the final grade, give multiple opportunities to replace or counter-balace scores on exams and also offer multiple extra credit opportunities to increase the final grade. No one has ever failed one of my 300-level classes.

#1 Linear Systems of Differential Equations, Trace-Determinant Plane, Bifurcation. VISUAL & ANALYTIC In this problem we are dealing with a matrix A whose entries depend on the value or a parameter α. The goal is to understand how important features of the matrix $A = \begin{bmatrix} \alpha & -\alpha/2 \\ 1 & -1 \end{bmatrix}$ change as α change, particularly thinking about its trajectory in the Trace-Determinant plane. You're given that the relationship between the trace T and the determinant D is $2D + T = -1$. By comparing this curve to the curve of the standard curve $D = T^2/4$ will allow one to tell what are the possible natures of the equilibrium point for A as α changes, since you will be in different sections of the TD-plane as α changes. If the nature of the equilibrium changes qualitatively for a particular value of α then you are dealing with a buifurcation value, and you should be able to sketch what the phase portrait look like before and after this bifurcation value is reached.

#2 Linearization, Hamiltonian function, Gradient function. ANALYTIC, VERBAL & VISUAL. This time you are given a non-linear system which has an unknown parameter α in it and asked whether it possesses a Hamiltonian function and a Gradient function. The system is $\dot{x} = \alpha x - y + y^3 = f(x, y; \alpha)$ and $\dot{y} = x + \alpha y + x^2 = g(x, y; \alpha)$. The condition on the existence of a Hamilton $H(x, y)$ is whether $f_x = -g_y$ at every point in the plane. The condition on the existence of a Gradient Function $G(x, y)$ is whether $f_y = g_x$ at every point in the plane. However both of these equations depend on α to determine whether they are true or not. If there exists an α value for which the statement is true *at every point in the plane*, then the answer is TRUE. For the third question one must decide what the behavior of the non-linear system looks near the origin, which involves linearizing the system and computing the Jacobian at $(0, 0)$ and extrapolating that the behavior of the system will resemble the behavior of the linearized system.

#3 Linear Systems of Differential Equations, Matrix Exponential, General Solution. ANALYTIC & VERBAL. The controversial third question! The point of this problem is for you to take what you know about the solution of linear systems with two eigenvalues and eigenvectors and re-write it using the peculiar-looking object called the matrix exponential. One thing to always remember: When someone asks you whether some given object is a solution of an IVP, it must pass two tests: a) the object must satisfy the given differential equation and b) the object must satisfy the given initial condition. If someone asks you to show two objects are identical, you need to set them equal to each other and see if one can produce a tautology (i.e., $0 = 0$). The third thing is that by the Existence and Uniqueness Theorem you know that solution trajectories in linear systems are **unique**. This means that these two "different" solutions MUST be two different ways of writing the same object. **(a)** Let the LHS=$\frac{d\vec{x}}{dt}$ and let the RHS= $A\vec{x}$ where $\vec{x}$ is the standard eignevctor solution $\vec{x} = c_1e^{\lambda_1 t}\vec{v_1} + c_2e^{\lambda_2 t}\vec{v_2}$. Show that LHS=RHS (HINT: the $\vec{v}_i$ are eigenvectors). Let LHS=$\vec{x}(0)$ (i.e. plug $t = 0$ into your eigenvector solution) and let RHS $= \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$. BY equating LHS=RHS you should end up with the given linear system. (HINT: Linear Systems have unique solutions when their matrix is invertible). **(c)** Given $\vec{x}(t) = e^{At}\vec{x}_0$ is a solution of an IVP that means that LHS= $\frac{d\vec{x}}{dt} = A\vec{x} =$ RHS (satisfies the DE) and LHS $= \vec{x}_0 = \vec{x}(0) =$ RHS (satisfies the initial condition at $t = 0$). **(d)** You need to show that the two objects, the general eigenvector solution and the matrix exponential solution are identical to each other, so set them equal to each other and either transofrm the LHS into the RHS or vice-versa in order to produce a tautology. In other words, start with $e^{At}\vec{x}_0 = c_1e^{\lambda_1 t}\vec{v_1} + c_2e^{\lambda_2 t}\vec{v_2}$ and us ethe information you are GIVEN about $\vec{v}_i$ as well as the information you are given about c_i from previous questions and what the format of e^{At} looks like to produce a tautology.