Report on Test 1 Prof. Ron Buckmire

Point Distribution (N=21)

Range	50+	46+	44+	43+	40+	39+	38+	35+	34+	33+	30+	29+	29-
Grade	A+	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Frequency	0	2	3	0	9	0	1	2	3	0	1	0	0

Summary The exam was designed to review the most important concepts in the course so far: Existence and Uniqueness theorem, the concept of a solution to an ODE, the ability to find equilibria and sketch bifurcations diagrams. Three short-answer (like reading quiz questions) were included with the clear instruction that you must EXPLAIN your answer choice in order to get the most credit. The questions were specifically designed so that were equally weighted towards students with different kinds of learning skills: verbal learners, visual learners and analytic (calculations-based) learners, and this designation noted on every question. However, overall, class performance was somewhat average. The mean score was 40.15(80%). The median score was 41 (82%). The high score was 49 (98%).

#1 Existence and Uniqueness Theorem, Functions, Equilibrium Solutions, Separation of Variables, Interval of Validity. ANALYTIC & VERBAL. In this probelm we are dealing with a sequence of initial value problems that look like $y' = y^2, y(0) = A$, where $A > 0$. **(a)** One of the basic ideas of Differential Equations is that if someone gives you a solution to an initial value problem, you can check whether they are correct or not. To do so, you either derive the solution yourself (which for this problem can be done using Separation of Variables) or you check that the differential equation **and** initial condition are satisfied by the given function, $y(t) = \dfrac{A}{1 - At}$. **(b)** If you're going to answer what the Existence and Uniqueness Theorem can allow you to say about the given initial problem then the first thing you need to do is STATE THE THEOREM (or at the very least, it's hypothesis and conclusions). Summarized, that would be that given $y' = f(t, y), \quad y(a) = b$ the Theorem says that IF $f(t, y)$ is continuous at (a, b) then $y(t)$ **exists** "near" (a, b) **and** IF $\frac{\partial f}{\partial y}$ is continuous at (a, b) then the solution $y(t)$ through (a, b) is **unique**. **(c)** This question is about what it means to be a solution of a differential equation. A differential equation (and it's solution) is only valid or well-defined on an interval $\mathcal{I}$, a connected subset of the real line. In most cases, the domain of the solution function and the interval of validity of the solution function are the same set of points but not always. Note: the location of the initial condition is always in the interval of validity for the differential equation. It turns out in this problem that the solution exhibits what is known as "finite time blow up." **(d)** Clearly, the interval of validity $\mathcal{I}$ must depend on A and the question is how. As A gets closer and closer to zero $\mathcal{I}$ changes. In the limit when $A = 0$ you have a new initial value problem $y' = y^2, y(0) = 0$. You should be able to solve this IVP independently (since it's an autonomous DE) and this solution's interval of validity should correspond to what you'd expect if you completed the limit as A decreased and tended towards zero while remaining positive.

#2 Slope Fields, Systems of DEs, Equilibria, Homogeneous and Non-homogeneous DEs. ANALYTIC, VERBAL & VISUAL (a) This question is asking about your knowledge of the differences between coupled and uncoupled (or decoupled) systems of differential equations. A good way to answer the problem is to provide examples of both. Note: decoupled equations do NOT have to be first order, autonomous DEs; they could be non-autonomous. **(b)** This question is taken directly from Reading Quiz 2. The quickest way to answer the question is to use operator representations of the ODE at hand. Another way is to pick a very simple counter-example like $y' = t$. **(c)** The given slopefield varies with both x and y because as one moves horizontally or vertically the slopes of the lines vary. However, there is clearly an equilibrium value at $y = 1$. There is no equilibrium value at $x = 0$, because equilibrium values are values of the dependent variable. This is a rare example of a non-autonomous DE having an equilibrium solution, similar to one we saw as a clicker question.

#3 Phase Lines, Equilibria, Bifurcations, Geometric Representations. VISUAL & ANALYTIC. This problem is about demonstrating your understanding of the meaning of bifurcation. Consider $y' = f(y; \alpha) = |y| + \alpha$. **(a)** The equilibria are values of y that depend on α such that $f(y^*) = 0$, i.e. $|y^*| = -\alpha$. But how can an absolute value equal $-\alpha$? Clearly, only when $\alpha = 0$ or $\alpha < 0$. When $|y^*| = -\alpha$ and α is negative, since y^* is in an absolute value there must be two possibilities, $-y^* = -\alpha$ and $y^* = -\alpha$. Thus the equilibria are at $y^* = \pm\alpha$ for $\alpha \leq 0$ **(b)** A bifurcation value is a value of the parameter α for which there is a change in the qualitative (kind of) or quantitative (number of) nature of equilibria. Clearly something happens when $\alpha = 0$ so this is the value α_B. It's impossible to use the analytic method of solving the simultaneous system $f(y; \alpha) = 0$ and $f_y(y; \alpha) = 0$ since y is not differentiable. In that case, you must use the visual method of considering graphs of $f(y)$ versus y for various values of α, or drawing the phase lines for various values of α (greater and less than $\alpha_B = 0$). **(c)** The bifurcation diagram is always just a vsiualization of the functional relationship between y^* and α which one obtains by solving $f(y; \alpha) = 0$ for the equilibrium values. In this case, that solution is $y^* = \pm\alpha$ for $\alpha \leq 0$, so that's the graph one should draw.