Differential Equations

Math 341 Fall 2009 MWF 2:30-3:25pm Fowler 110
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Worksheet 9: **Friday September 30**

TITLE Integrating Factors
CURRENT READING Blanchard, 1.9

Homework Assignments due Friday October 2
Section 1.8: 4, 5, 8, 9, 17, 18, 20.
Section 1.9: 4, 5, 9, 12, 19.

SUMMARY
We will learn a very cool analytical technique for obtaining a formula for solutions of linear ODEs.

Consider re-writing the standard linear DE $\frac{dy}{dx} = a(x)y + b(x)$ as

$$\frac{dy}{dx} + P(x)y = Q(x) \tag{1}$$

EXAMPLE **Integrating Factor**

It turns out that if one takes the function $\mu(x) = e^{\int P(x)dx}$ and multiplies each term in the modified standard form in (1) by this integrating factor one obtains:

$$\begin{aligned}
e^{\int P(x)dx}\frac{dy}{dx} + e^{\int P(x)dx}P(x)y &= e^{\int P(x)dx}Q(x) \\
\frac{d}{dx}\left(e^{\int P(x)dx}y\right) &= Q(x)e^{\int P(x)dx} \\
e^{\int P(x)dx}y &= \int Q(x)e^{\int P(x)dx}\,dx \\
y(x) &= e^{-\int P(x)dx}\int Q(x)e^{\int P(x)dx}\,dx
\end{aligned}$$

This is an exact formula for general solutions to the equation in (1).

EXAMPLE

Solve $\frac{dy}{dt} = -2ty + 4e^{-t^2}$

Exercise

Blanchard, page 135, Question 7. Solve $\frac{dy}{dt} = -\frac{y}{1+t} + 2 \quad y(0) = 3.$

GroupWork

Blanchard, page 135, Question 20. For what value(s) of the parameter r is it possible to find explicit formulas (without integrals) for the solution to

$$\frac{dy}{dt} = t^r y + 4?$$