

1. Consider the slope field for the given system

$$A = \begin{pmatrix} -2 & 1/2 \\ 0 & -1 \end{pmatrix}$$

$$\begin{aligned} \frac{dx}{dt} &= -2x + \frac{1}{2}y \\ \frac{dy}{dt} &= -y \end{aligned}$$

$$\begin{vmatrix} -2-\lambda & 1/2 \\ 0 & -1-\lambda \end{vmatrix} = 0$$

$$\lambda = -1$$

$$\begin{pmatrix} -1 & 1/2 & : & 0 \\ 0 & 0 & : & 0 \end{pmatrix}$$

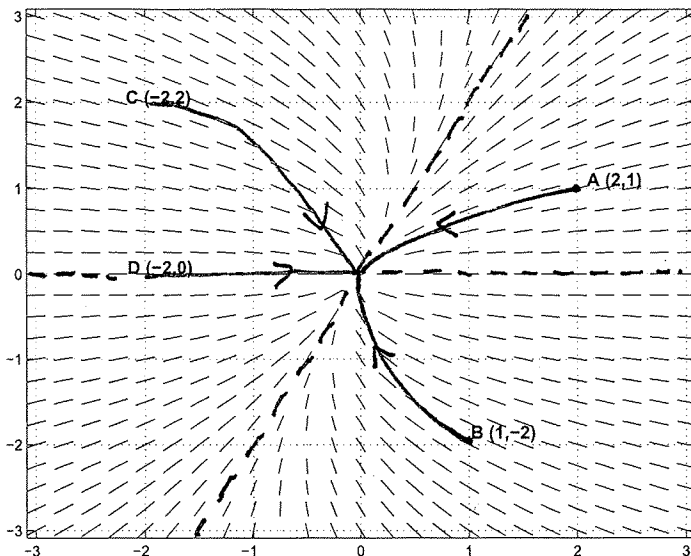
$$\vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\lambda = -2$$

$$\begin{pmatrix} 0 & 1/2 & : & 0 \\ 0 & 1 & : & 0 \end{pmatrix} \rightarrow$$

$$\vec{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} (-2-\lambda)(-1-\lambda) &= 0 \\ (2+\lambda)(1+\lambda) &= 0 \\ \lambda &= -2 \\ \lambda &= -1 \end{aligned}$$



(a) 2 points. Determine the type of equilibrium point at the origin.

stable ~~im~~ proper node  
 $0 > \lambda_1 > \lambda_2$   
 $(0,0)$  is an attractive stationary point

(b) 4 points. Indicate the trajectories for solutions which start at the initial conditions  $A = (2, 1)$ ,  $B = (1, -2)$ ,  $C = (-2, 2)$  and  $D = (-2, 0)$ . (USE ARROWS!)

(c) 4 points. In the space, sketch graphs of  $x(t)$  and  $y(t)$  on the same axis for each of the given four initial conditions. (Therefore you should have four pairs of axes, with 8 curves.) Clearly indicate what happens as  $t \rightarrow \infty$  for each solution.

