Math 341 Fall 2009 **SHOW ALL YOUR WORK** Quiz **Two**

1. Consider the following one-parameter family of nonlinear, first-order, autonomous differential equations where α is a known real parameter value

$$\frac{dy}{dx} = y^2 - \alpha y + 1.$$

(a) *2 points.* Show that this DE has no equilibrium values when $|\alpha| < 2$.

$$y^2 - \alpha y + 1 = 0$$

$$y^* = \frac{\alpha \pm \sqrt{(-\alpha)^2 - 4\cdot 1\cdot 1}}{2\cdot 1} = \frac{\alpha \pm \sqrt{\alpha^2-4}}{2}$$

y^* will have real solutions when $-2 < \alpha < 2$

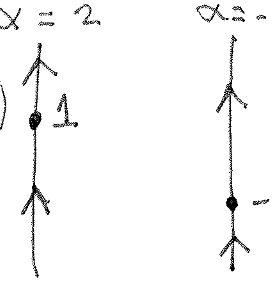
$\alpha^2 - 4 \geq 0 \Rightarrow \alpha^2 \geq 4 \Rightarrow \alpha \geq 2$ or $\alpha \leq -2$

(b) *2 points.* For what values of α will the DE have exactly one equilibrium value? Classify the equilibrium point (as node, source or sink) in this case and write down the constant solution.

$\alpha^2 - 4 = 0$

$\alpha^2 = 4$

$\alpha = \pm 2$ (node)



(c) *4 points.* Show that when $|\alpha| > 2$ the DE has exactly one stable equilibrium value (sink) and one unstable equilibrium value (source). Show work that supports your classification of the equilibria, and sketch a phase line for a representative value of α.

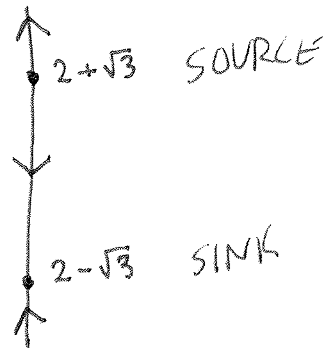
$\alpha = 4$

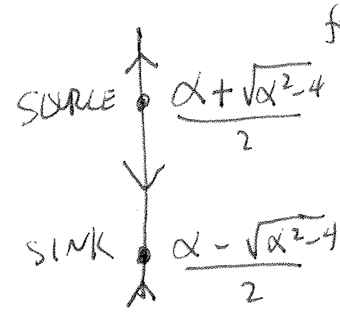
$$y^* = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$$

$f(y;\alpha) = y^2 - \alpha y + 1$

$f'(y) = 2y - \alpha = \pm\sqrt{\alpha^2-4}$

Thus there will always be one source and one sink for every $|\alpha| > 2$





(d) *2 points.* Use your answers from above to sketch the bifurcation diagram for the given DE. Clearly indicate where sources, sinks and nodes occur.