Math 341 Fall 2009

BONUS QUIZ 2 **Differential Equations**

Name: ________________________________

Friday October 30
Prof. Ron Buckmire

Topic : Visualizing Solutions of Linear Systems of ODEs

The idea behind this quiz is to provide you with an opportunity to illustrate your understanding of solution techniques for systems of n linear ordinary differential equations.

Reality Check:

EXPECTED SCORE : __________**/5** ACTUAL SCORE : __________**/5**

Instructions:

0. Please look for a hint on this quiz posted to `faculty.oxy.edu/ron/math/341/09/`
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This bonus quiz is due on Monday November 2**, at the beginning of class.
   NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, ______________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. Consider the system of ordinary differential equations

$$\frac{d\vec{x}}{dt} = A\vec{x} = \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix} \vec{x} \text{ where } \vec{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

(a) *1 point.* Show that the matrix A has eigenvalues 0 and -1 and eigenvectors which are multiples of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$. Write down the 2-parameter general solution of the system $\dfrac{d\vec{x}}{dt} = A\vec{x}$.

(b) *2 points.* Find the exact solution $\vec{x}(t)$ for each of the trajectories which go through the points $\mathbf{A}(1,1)$, $\mathbf{B}$ $(0,-2)$ and $\mathbf{C}$ $(4,0)$ at $t = 0$.

(c) *2 points.* On the figure below clearly indicate the trajectories for each of the solutions which start at $\mathbf{A}(1,1)$, $\mathbf{B}$ $(0,-2)$ and $\mathbf{C}$ $(4,0)$ ends up as $t \to \infty$. Label these endpoints $\mathbf{A}'$, $\mathbf{B}'$ and $\mathbf{C}'$ respectively.