Math 341 Spring 2005 **SHOW ALL YOUR WORK** Quiz Seven

1. Consider the system of ordinary differential equations

$$\frac{d\vec{x}}{dt} = A\vec{x} = \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix} \vec{x} \text{ where } \vec{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

(a) *4 points.* Show that the matrix A has eigenvalues 0 and -1 and eigenvectors which are multiples of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$. Write down the general solution of the system.

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 2 \\ 0 & -1-\lambda \end{vmatrix} = \lambda(1+\lambda) = 0 \Rightarrow \lambda = 0 \text{ or } -1$$

$\lambda = 0$

$\left(\begin{array}{cc|c} 0 & 2 & 0 \\ 0 & -1 & 0 \end{array}\right) \Rightarrow \vec{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\qquad \vec{x}(t) = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-t}$

$\lambda = -1$

$\begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} \Rightarrow \vec{x} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

(b) *3 points.* Find the exact solution for each of the trajectories which go through the points A $(1,1)$, B $(0,-2)$ and C $(4,0)$.

$A \begin{pmatrix} 1 \\ 1 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} \Rightarrow \begin{matrix} c_2 = 1 \\ c_1 = 3 \end{matrix} \quad \vec{x}_A(t) = \begin{pmatrix} 3 - 2e^{-t} \\ e^{-t} \end{pmatrix}$

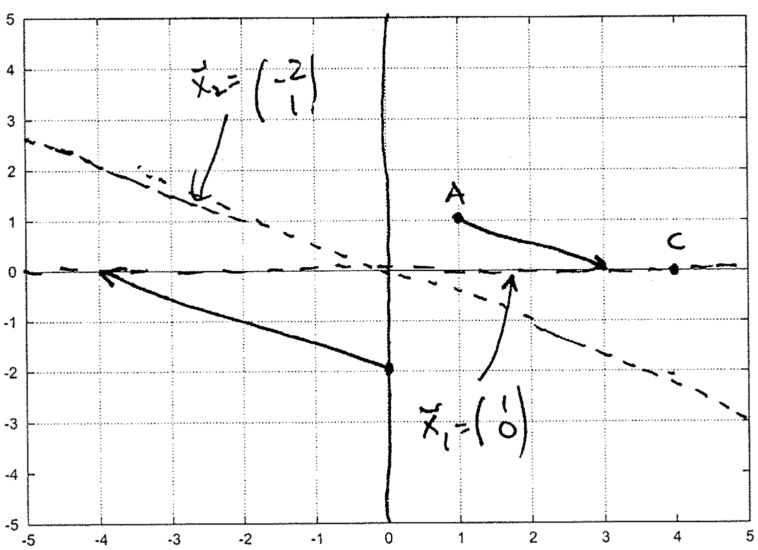
$B \begin{pmatrix} 0 \\ -2 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} \Rightarrow \begin{matrix} c_2 = -2 \\ c_1 = -4 \end{matrix} \quad \vec{x}_B(t) = \begin{pmatrix} -4 + 4e^{-t} \\ -2e^{-t} \end{pmatrix}$

$C \begin{pmatrix} 4 \\ 0 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} -2 \\ 1 \end{pmatrix} \Rightarrow \begin{matrix} c_2 = 0 \\ c_1 = 4 \end{matrix} \quad \vec{x}_C(t) = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$\frac{dx}{dt} = 2y \quad \frac{dy}{dt} = -y \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-y}{+2y} = -\frac{1}{2} = \frac{dy}{dx} \leftarrow$ constant slope

(c) *3 points.* On the figure below clearly indicate where each of the trajectories of the solutions which start at $(1,1)$, $(0,-2)$ and $(4,0)$ ends up as $t \to \infty$.

$y = 0$ is equilibrium (stable) solution of system



As $t \to \infty$

$x_A \to \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$x_B \to \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

x_C always at $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$