Multiplying both sides by $\mu(t)$, we obtain

$$(1 + t) \frac{dy}{dt} + y = (1 + t)t^2.$$  

Applying the Product Rule to the left-hand side, we see that this equation is the same as

$$\frac{d((1 + t)y)}{dt} = t^3 + t^2,$$

and integrating both sides with respect to $t$, we obtain

$$(1 + t)y = \frac{t^4}{4} + \frac{t^3}{3} + c,$$

where $c$ is an arbitrary constant. The general solution is

$$y(t) = \frac{3t^4 + 4t^3 + 12c}{12(t + 1)}.$$

4. We rewrite the equation in the form

$$\frac{dy}{dt} + 2ty = 4e^{-t^2}$$

and note that the integrating factor is

$$\mu(t) = e^{\int 2t \, dt} = e^{t^2}.$$  

Multiplying both sides by $\mu(t)$, we obtain

$$e^{t^2} \frac{dy}{dt} + 2te^{t^2} y = 4.$$

Applying the Product Rule to the left-hand side, we see that this equation is the same as

$$\frac{d(e^{t^2} y)}{dt} = 4,$$

and integrating both sides with respect to $t$, we obtain

$$e^{t^2} y = 4t + c,$$

where $c$ is an arbitrary constant. The general solution is

$$y(t) = 4te^{-t^2} + ce^{-t^2}.$$
5. Note that the integrating factor is
\[ \mu(t) = e^{\int (-2/(1+t^2)) \, dt} = e^{-\ln(1+t^2)} = (e^{\ln(1+t^2)})^{-1} = \frac{1}{1+t^2}. \]

Multiplying both sides by \( \mu(t) \), we obtain
\[ \frac{1}{1+t^2} \frac{dy}{dt} - \frac{2t}{(1+t^2)^2} y = \frac{3}{1+t^2}. \]

Applying the Product Rule to the left-hand side, we see that this equation is the same as
\[ \frac{d}{dt} \left( \frac{y}{1+t^2} \right) = \frac{3}{1+t^2}. \]

Integrating both sides with respect to \( t \), we obtain
\[ \frac{y}{1+t^2} = 3 \arctan(t) + c, \]
where \( c \) is an arbitrary constant. The general solution is
\[ y(t) = (1+t^2)(3 \arctan(t) + c). \]

6. Note that the integrating factor is
\[ \mu(t) = e^{\int (-2/t) \, dt} = e^{-2\ln t} = e^{\ln(t^{-2})} = t^{-2}. \]

Multiplying both sides by \( \mu(t) \), we obtain
\[ t^{-2} \frac{dy}{dt} - 2t^{-3} y = te^t. \]

Applying the Product Rule to the left-hand side, we see that this equation is the same as
\[ \frac{d(t^{-2}y)}{dt} = te^t, \]
and integrating both sides with respect to \( t \), we obtain
\[ t^{-2} y = (t-1)e^t + c, \]
where \( c \) is an arbitrary constant. The general solution is
\[ y(t) = t^2(t-1)e^t + ct^2. \]
Integrating both sides with respect to \( t \), we obtain

\[
\frac{y}{t + 1} = 2t^2 + c,
\]

where \( c \) is an arbitrary constant. The general solution is

\[
y(t) = (2t^2 + c)(t + 1) = 2t^3 + 2t^2 + ct + c.
\]

To find the solution that satisfies the initial condition \( y(1) = 10 \), we evaluate the general solution at \( t = 1 \) and obtain \( c = 3 \). The desired solution is

\[
y(t) = 2t^3 + 2t^2 + 3t + 3.
\]

9. In Exercise 1, we derived the general solution

\[
y(t) = t + \frac{c}{t}.
\]

To find the solution that satisfies the initial condition \( y(1) = 3 \), we evaluate the general solution at \( t = 1 \) and obtain \( c = 2 \). The desired solution is

\[
y(t) = t + \frac{2}{t}.
\]

10. In Exercise 4, we derived the general solution

\[
y(t) = 4te^{-t^2} + ce^{-t^2}.
\]

To find the solution that satisfies the initial condition \( y(0) = 3 \), we evaluate the general solution at \( t = 0 \) and obtain \( c = 3 \). The desired solution is

\[
y(t) = 4te^{-t^2} + 3e^{-t^2}.
\]

11. Note that the integrating factor is

\[
\mu(t) = e^{\int -\frac{2}{t} \, dt} = e^{-2\int \frac{1}{t} \, dt} = e^{-2\ln t} = e^{\ln(t^{-2})} = \frac{1}{t^2}.
\]

Multiplying both sides by \( \mu(t) \), we obtain

\[
\frac{1}{t^2} \frac{dy}{dt} - \frac{2y}{t^3} = 2.
\]

Applying the Product Rule to the left-hand side, we see that this equation is the same as

\[
\frac{d}{dt} \left( \frac{y}{t^2} \right) = 2,
\]
and integrating both sides with respect to $t$, we obtain

$$\frac{y}{t^2} = 2t + c,$$

where $c$ is an arbitrary constant. The general solution is

$$y(t) = 2t^3 + ct^2.$$

To find the solution that satisfies the initial condition $y(-2) = 4$, we evaluate the general solution at $t = -2$ and obtain

$$-16 + 4c = 4.$$

Hence, $c = 5$, and the desired solution is

$$y(t) = 2t^3 + 5t^2.$$

12. Note that the integrating factor is

$$\mu(t) = e^{\int \frac{1}{t} dt} = e^{-\ln t} = e^{\ln t^{-1}} = t^{-3}.$$

Multiplying both sides by $\mu(t)$, we obtain

$$t^{-3} \frac{dy}{dt} - 3t^{-4} y = 2e^{2t}.$$

Applying the Product Rule to the left-hand side, we see that this equation is the same as

$$\frac{d(t^{-3}y)}{dt} = 2e^{2t},$$

and integrating both sides with respect to $t$, we obtain

$$t^{-3}y = e^{2t} + c,$$

where $c$ is an arbitrary constant. The general solution is

$$y(t) = t^{3}(e^{2t} + c).$$

To find the solution that satisfies the initial condition $y(1) = 0$, we evaluate the general solution at $t = 1$ and obtain $c = -e^2$. The desired solution is

$$y(t) = t^{3}(e^{2t} - e^2).$$

13. We rewrite the equation in the form

$$\frac{dy}{dt} - (\sin t)y = 4$$

and note that the integrating factor is

$$\mu(t) = e^{\int (-\sin t) dt} = e^{\cos t}.$$
and integrating both sides with respect to \( t \), we obtain
\[
\left( e^{-\int \frac{1}{\sqrt{t^3-3}} \, dt} \right) y = \int t \left( e^{-\int \frac{1}{\sqrt{t^3-3}} \, dt} \right) \, dt.
\]

These integrals are also impossible to express in terms of elementary functions, so we write the general solution in the form
\[
y(t) = \left( e^{\int \frac{1}{\sqrt{t^3-3}} \, dt} \right) \int t \left( e^{-\int \frac{1}{\sqrt{t^3-3}} \, dt} \right) \, dt.
\]

**19.** We rewrite the equation in the form
\[
\frac{dy}{dt} - aty = 4e^{-t^2}
\]
and note that the integrating factor is
\[
\mu(t) = e^{\int (-at) \, dt} = e^{-at^2/2}.
\]

Multiplying both sides by \( \mu(t) \), we obtain
\[
e^{-at^2/2} \frac{dy}{dt} - ate^{-at^2/2} y = 4e^{-t^2} e^{-at^2/2}.
\]

Applying the Product Rule to the left-hand side and simplifying the right-hand side, we see that this equation is the same as
\[
\frac{d(e^{-at^2/2} y)}{dt} = 4e^{-(1+a/2)t^2}.
\]

Integrating both sides with respect to \( t \), we obtain
\[
e^{-at^2/2} y = \int 4e^{-(1+a/2)t^2} \, dt.
\]

The integral on the right-hand side can be expressed in terms of elementary functions only if \( 1 + a/2 = 0 \) (that is, if the factor involving \( e^{t^2} \) really isn’t there). Hence, the only value of \( a \) that yields an integral we can express in terms of elementary functions form is \( a = -2 \) (see Exercise 4).

**20.** We rewrite the equation in the form
\[
\frac{dy}{dt} - tr^y = 4
\]
and note that the integrating factor is
\[
\mu(t) = e^{-\int tr \, dt}.
\]

There are two cases to consider.

(a) If \( r \neq -1 \), then
\[
\mu(t) = e^{-t^{r+1}/(r+1)}.
\]

Multiplying both sides of the differential equation by \( \mu(t) \), we obtain
\[
\left( e^{-t^{r+1}/(r+1)} \right) \frac{dy}{dt} - tr^{r} \left( e^{-t^{r+1}/(r+1)} \right) y = 4 \left( e^{-t^{r+1}/(r+1)} \right).
\]