3. \[ \frac{dP}{dt} = 0.4P \left( 1 - \frac{P}{230} \right) \]

(a) Equilibrium occurs when \( \frac{dP}{dt} = 0 \)

\[ P = 0 \text{ or } P = 230 \]

(b) \( P \uparrow \) when \( \frac{dP}{dt} > 0 \)

\[ P \left( 1 - \frac{P}{230} \right) > 0 \]

\[ P(230 - P) > 0 \]

\[ 0 < P < 230 \]

(c) \( P \downarrow \) means \( P' < 0 \)

\[ P \left( 1 - \frac{P}{230} \right) < 0 \]

\[ P > 230 \text{ or } P < 0 \text{ not really possible.} \]

4. \[ \frac{dP}{dt} = 0.3 \left( 1 - \frac{P}{200} \right) \left( \frac{P}{50} - 1 \right) P \]

(a) \( P = 0 \text{ or } P = 200 \text{ or } P = 50 \) cause \( \frac{dP}{dt} = 0 \)

(b) \( \frac{dP}{dt} < 0 \) when \( 0 < P < 50 \text{ or } P > 200 \)

(c) \( \frac{dP}{dt} > 0 \) when \( P < 0 \) (non-physical)

\[ 50 < P < 200 \]
8. \[ \frac{dl}{dt} = 2(1-L) \]

At \( t=0 \), \( L_1 = \frac{1}{2} \) \( \Rightarrow \frac{dl_1}{dt} = 2 \left( 1 - \frac{1}{2} \right) = 1 \)

At \( t=0 \), \( L_2 = 0 \) \( \Rightarrow \frac{dl_2}{dt} = 2 \left( 1 - 0 \right) = 2 \)

\( L_2'(0) > L_1'(0) \)

So the rate of learning of the second student is faster.

11. \[ \frac{dr}{dt} = -\lambda r, \quad r(0) = r_0 \Rightarrow r(t) = r_0 e^{-\lambda t} \]

(a) For Carbon-14, \( r(5230) = \frac{1}{2} r_0 \)

\[ \frac{r_0}{2} = r_0 e^{-\lambda 5230} \]

\[ \frac{1}{2} = e^{-\lambda 5230} \]

\[ \ln \left( \frac{1}{2} \right) = -\lambda 5230 \]

\[ -\ln 2 = -\lambda 5230 \]

\[ \frac{\ln 2}{5230} = \lambda \]

(b) For Iodine-131

\[ \lambda = \frac{\ln 2}{8} \]

(c) Units of \( \lambda \) are \((\text{time}^{-1})\) or \(\text{year}^{-1}\) per year or \(\text{day}^{-1}\) per day.

(d) It always takes the same time to decay to "half" the size of the initial sample, called the "half-life."