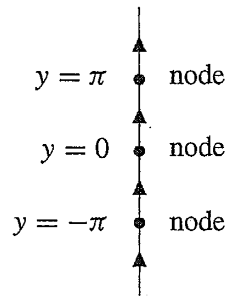
2. By guessing or separating variables, we know that the general solution is $y(t) = y_0 e^{3t}$, where $y(0) = y_0$ is the initial condition.

3. There are no values of y for which dy/dt is zero for all t. Hence, there are no equilibrium solutions.

4. Since the question only asks for one solution, look for the simplest first. Note that $y(t) = 0$ for all t is an equilibrium solution. There are other equilibrium solutions as well.

5. The right-hand side is zero for all t only if $y = -1$. Consequently, the function $y(t) = -1$ for all t is the only equilibrium solution.

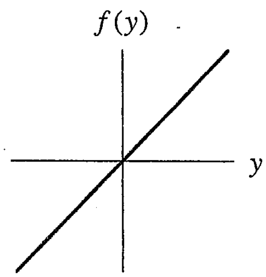
6. The equilibria occur at $y = \pm n\pi$ for $n = 0, 1, 2, \ldots$, and dy/dt is positive otherwise. So all of the arrows between the equilibrium points point up.



7. The equations $dy/dt = y$ and $dy/dt = 0$ are first-order, autonomous, separable, linear, and homogeneous.

8. The equation $dy/dt = y - 2$ is autonomous, linear, and nonhomogeneous. Moreover, if $y = 2$, then $dy/dt = 0$ for all t.

9. The graph of $f(y)$ must cross the y-axis from negative to positive at $y = 0$. For example, the graph of the function $f(y) = y$ produces this phase line.



10. For $a > -4$, all solutions increase at a constant rate, and for $a < -4$, all solutions decrease at a constant rate. Consequently, a bifurcation occurs at $a = -4$, and all solutions are equilibria.

11. True. We have $dy/dt = e^{-t}$, which agrees with $|y(t)|$.

12. False. A separable equation has the form $dy/dt = g(t)h(y)$. So if $g(t)$ is not constant, then the equation is not separable. For example, $dy/dt = ty$ is separable but not autonomous.

28. **(a)** This equation is linear and nonhomogeneous.

(b) First we note that the general solution of the associated homogeneous equation is ke^{-3t}.

Next we use the technique suggested in Exercise 19 of Section 1.8. We could find particular solutions of the two nonhomogeneous equations

$$\frac{dy}{dt} = -3y + e^{-2t} \quad \text{and} \quad \frac{dy}{dt} = -3y + 4$$

separately and add the results to obtain a particular solution for the original equation. However, these two steps can be combined by making a more complicated guess for the particular solution.

We guess $y_p(t) = ae^{-2t} + b$, and we have

$$\begin{aligned} \frac{dy_p}{dt} + 3y_p &= -2ae^{-2t} + 3ae^{2t} + 3b \\ &= ae^{-2t} + 3b. \end{aligned}$$

Hence, for $y_p(t)$ to be a solution we must have $a = 1$ and $b = 4/3$. Therefore, a particular solution is $y_p(t) = e^{-2t} + \frac{4}{3}$, and the general solution is

$$y(t) = ke^{-3t} + e^{-2t} + \tfrac{4}{3}.$$

29. **(a)** The equation is linear and nonhomogeneous. (It is nonautonomous as well.)

(b) The general solution to the associated homogeneous equation is $y_h(t) = ke^{-t}$. For a particular solution of the nonhomogeneous equation, we guess a solution of the form $y_p(t) = at^2+bt+c$. Then we must have

$$\frac{dy_p}{dt} + y_p = t^2.$$

In other words, we must have $(2at + b) + (at^2 + bt + c) = t^2$ for $y_p(t)$ to be a solution. Rewriting the left-hand side, we get

$$at^2 + (2a + b)t + (b + c) = t^2,$$

so we must solve

$$\begin{cases} a = 1 \\ 2a + b = 0 \\ b + c = 0. \end{cases}$$

Hence, $a = 1$, $b = -2$, and $c = 2$, and our particular solution is

$$y_p(t) = t^2 - 2t + 2.$$

The general solution is $y(t) = ke^{-t} + t^2 - 2t + 2$.

30. **(a)** This equation is separable and autonomous.

(b) First, note that $y = 0$ and $y = 2$ are the equilibrium points. Assuming that $y \neq 0$ and $y \neq 2$, we separate variables to obtain

$$\int \frac{1}{2y - y^2}\, dy = \int dt.$$

47. **(a)** Note that there is an equilibrium solution of the form $y = -1/2$.
Separating variables and integrating, we obtain

$$\int \frac{1}{2y+1}\,dy = \int \frac{1}{t}\,dt$$

$$\tfrac{1}{2}\ln|2y+1| = \ln|t| + c$$

$$\ln|2y+1| = (\ln t^2) + c$$

$$|2y+1| = c_1 t^2,$$

where $c_1 = e^c$. We can eliminate the absolute value signs by allowing the constant to be either positive or negative. In other words, $2y + 1 = k_1 t^2$, where $k_1 = \pm c_1$. Hence

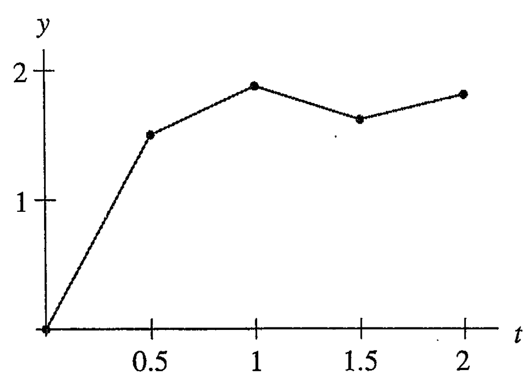
$$y(t) = kt^2 - \tfrac{1}{2},$$

where $k = k_1/2$.

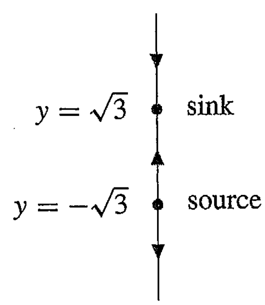
(b) As t approaches zero all the solutions approach $-1/2$. In fact, $y(0) = -1/2$ for every value of k.

(c) This example does not violate the Uniqueness Theorem because the differential equation is not defined at $t = 0$. So functions $y(t)$ can only be said to be solutions for $t \neq 0$.

48. **(a)** Using Euler's method, we obtain the values $y_0 = 0$, $y_1 = 1.5$, $y_2 = 1.875$, $y_3 = 1.617$, and $y_4 = 1.810$ (rounded to three decimal places).



(b)



(c) The phase line tells us that the solution with initial condition $y(0) = 0$ must be increasing. Moreover, its graph is below and asymptotic to the line $y = \sqrt{3}$ as $t \to \infty$. The oscillations obtained using Euler's method come from numerical error.

49. **(a)** If we let k denote the proportionality constant in Newton's law of cooling, the initial-value problem satisfied by the temperature T of the soup is

$$\frac{dT}{dt} = k(T - 70), \quad T(0) = 150.$$