Differential Equations

Math 341 Fall 2008 © 2008 Ron Buckmire $MWF~2:30\text{-}3:25pm~Fowler~307\\ \texttt{http://faculty.oxy.edu/ron/math/341/08/}$

Class 4: Friday September 5

TITLE Euler's Method

CURRENT READING Blanchard, 1.4

Homework Assignments due Friday September 12

Section 1.4: 5, 6, 13, 15

Section 1.5: 2, 3, 12, 14, 15.

Section 1.6: 2, 7, 8, 19, 20, 30, 31, 41

SUMMARY

We will learn about a simple (and possibly familiar) numerical technique called Euler's Method which approximates solutions to ODEs quantitatively.

Euler's Method

Given an expression for how the derivative of an unknown function y(x) changes, i.e. y' = f(x, y), and an initial value $y(x_0) = y_0$ one can use Euler's Method to estimate y(x) at a point close by with bounded error.

$$y(x_{new}) = y(x_{old}) + \Delta y$$
 where $\Delta y \approx y'(x_{old}) \Delta x$

In other words

 $y_{new} \approx y_{old} + y'_{old} \Delta x$ and $x_{new} = x_{old} + \Delta x$ or $y_{k+1} \approx y_k + f(x_k, y_k) \Delta x$ and $x_{k+1} = x_k + \Delta x$. **Exercise**

Draw a picture of the Euler approximation y_{new} starting at a point (x_{old}, y_{old}) . Is the slope field involved?

EXAMPLE

Using Euler's Method To Approximate Solutions To Differential Equations

- 1. Suppose y changes with time t according to the equation y' = 1 + 2y.
- (a) What is the rate of change of y when y = 3?
- (b) Suppose when t = 0, y = 3. Use Euler's Method with $\Delta t = .5$ to estimate y(1).
- (c) Is your estimate of y(1) an over-estimate or under-estimate?

To use Euler's Method generally the following table can be helpful

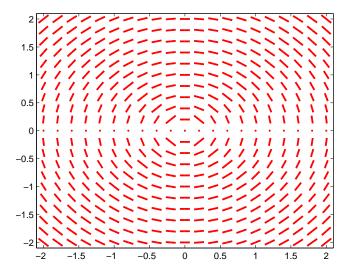
x	y	y'	Δy

Slope Fields and Euler's Method

Exercise

Consider the differential equation y' = -x/y with initial condition y(0) = 1. Given that the exact solution is $y(x) = \sqrt{1 - x^2}$,

- (a) use the slope field to estimate y(1/2) for the solution that satisfies the given initial condition.
- (b) Compare your estimate with the exact value of y(1/2)
- (c) Use Euler's Method with $\Delta x = .25$ to estimate y(1/2).
- (d) Is your Euler's Method estimate and over-estimate or under-estimate? Explain why.



To use Euler's Method generally the following table can be helpful

x	y	y'	Δy

Numerical Error in Using Euler's Method

GROUPWORK

Complete the following sentences:	
As the time step Δt	in magnitude, the numerical error in computing $y(x_0)$
using Euler's Method decreases in	n magnitude.
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using Euler's Method increases in	magnitude.
When y'' is	on $x_0 < x < x_1$ the function $y(x)$ is con-
cave up and estimates of $y(x_1)$ usin	ag Euler's Method will be
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