Differential Equations

Math 341 Fall 2008 MWF 2:30-3:25pm Fowler 307
 http://faculty.oxy.edu/ron/math/341/08/

Class 3: Wednesday September 3

TITLE Analyzing DEs Using Slope Fields
CURRENT READING Blanchard, 1.2 and 1.3

Homework Assignments due Friday September 5
Section 1.2: 1, 2, 3, 6, 25, 27, 32.
Section 1.3: 7, 8, 9, 10, 12, 15.

SUMMARY
We begin our analysis of first order DEs by discovering how much information about the unknown solution one can obtain without being able to obtain an explicit formula for the solution itself by using a technique called a **slope field**.

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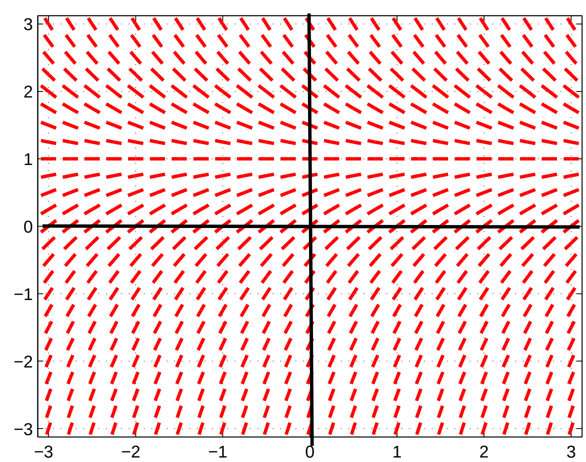
1. Slope Fields

DEFINITION: **direction field** or **slope field**
A collection of short, oriented line segments called lineal elements placed at each point (x, y) over a rectangular grid , which have slopes evaluated at each point to be $f(x, y)$ are called the **direction field** or **slope field** of the first order differential equation $\dfrac{dy}{dx} = f(x, y)$.

EXAMPLE
Let's consider the slope field $y' = 1 - y$

At every point in the xy-plane there is a little line representing a potential tangent or slope to the unknown function $y(x)$. The solution to the differential equation will start at the point in the plane reprepsented by the initial condition, i.e. $x = 0, y = -1$ and move tangentially to each of the little lines. Another way to think of the slope field is to view it as a still picture of running water with flecks of dirt in it and the solution curve will follow the path of the flecks of dirt.



There are numerous interesting things we can notice about the slope field for $y' = 1 - y$:
1) All the little lines are angled the same way as we move from left to right on a horizontal path, in other words y' does not depend on x.
2) When $y = 1$ all the little lines are horizontal. What does that mean?
3) When $y > 1$ all solution curves appear to be decreasing and concave up
4) When $y < 1$ all solution curves appear to be increasing and concave down

We could have predicted all of these features of the slope field from analyzing the differential equation $y' = 1 - y$

Observation	**Explanation from Analysis of DE**
1) y' does not depend on x	
2) When $y = 1$ all slopes horizontal	
3) When $y > 1$ all solutions $\downarrow$ and $\cup$	
4) When $y < 1$ all solutions $\uparrow$ and $\cap$	

$\boxed{\text{GroupWork}}$

Anton, Bivens & Davis, Page 601, Number 9. Match the differential equations with their corresponding slope fields.

(a) $y' = 1/x$ **(b)** $y' = 1/y$ **(c)**$y' = e^{-x^2}$

(d) $y' = y^2 - 1$ **(e)** $y' = \dfrac{x+y}{x-y}$ **(f)**$y' = \sin(x)\sin(y)$

FIGURE I

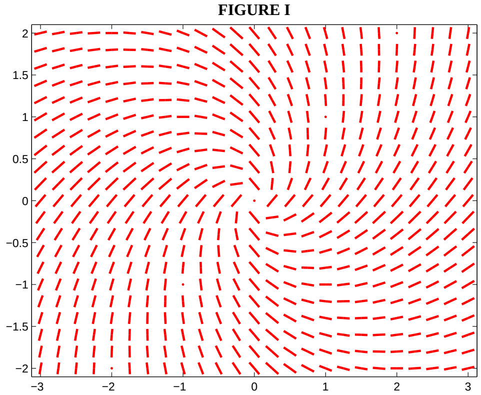
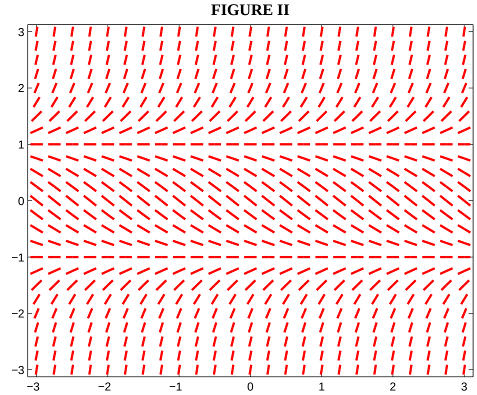


FIGURE II



FIELD III

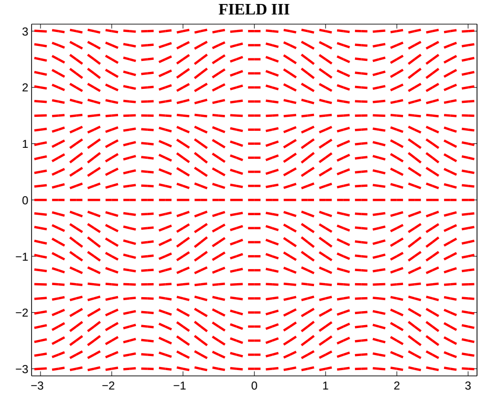
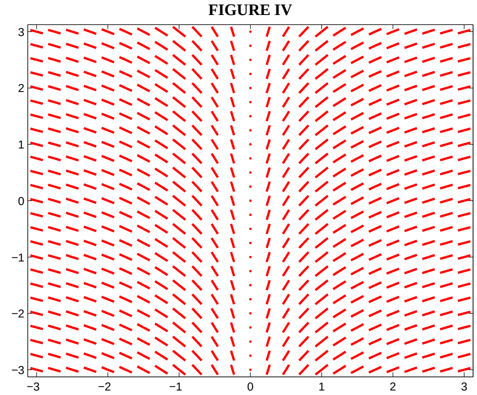


FIGURE IV



FIELD V

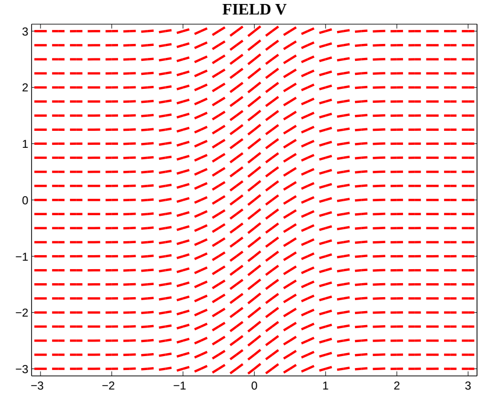


FIGURE VI