Differential Equations

Math 341 Fall 2008 ©2008 Ron Buckmire MWF 2:30-3:25pm Fowler 307 http://faculty.oxy.edu/ron/math/341/08/

Class 2: Friday August 29

TITLE Separation of Variables **CURRENT READING** Blanchard, §1.2 and §1.3

Homework Assignments due Friday September 5 Section 1.2: 1, 2, 3, 6, 25, 27, 32. Section 1.3: 7, 8, 9, 10, 12, 15.

SUMMARY

In today's class we shall review an analytical technique for solving a particular class of first-order (separable) ODEs known as **Separation of Variables**.

1. Solving Separable Differential Equations

DEFINITION: separable DE

A separable first-order differential equation is one which has the form $\frac{dy}{dx} = g(x)h(y)$

The technique for solution is to separate the variables in the equation by placing everything with an independent variable on one side, and everything with a dependent variable on the other. This produces:

$$\frac{dy}{h(y)} = g(x)dx$$

One can then treat each side of the equation as an indefinite integral,

$$\int \frac{dy}{h(y)} = \int g(x)dx$$

which, if each function 1/h(y) and g(x) have anti-derivatives H(y) and G(x), respectively produces

$$H(y) = G(x) + C$$

The above equation thus defines (implicitly) a family of solutions to the given first-order DE. When an initial condition y(a) = b is also given, then a particular solution can be obtained.

EXAMPLE

Let's consider the Malthusian Model of population $P' = kP, P(0) = P_0$ and obtain the solution by separation of variables.

Exercise

Let's consider the Verhulst or Logistic Model of Population $P' = kP(1 - P/N), P(0) = P_0$. Can you obtain a solution by the method of separation of variables?