MATH 341: DIFFERENTIAL EQUATIONS — Reading Quiz **FIVE** (Sections 6.1-6.5 of Blanchard)
November 21, 2008 — *Prof. R. Buckmire*

Your Name: BUCKMIRE **SCORE:** /10

INSTRUCTIONS: Answer the following short-answer questions (in 10-15 minutes).
GOAL: This reading quiz is designed to illuminate your understanding of the main concepts in Chapter 6 of the textbook: Laplace Transforms, Inverse Laplace Transforms, The Heaviside Function, the Dirac Delta Function, Laplace Translation Theorems and Convolutions. (NOTE: the Laplace Transforms Help Sheet on the reverse of this page!)

1. (*2 points.*) If such a formula exists, write down a general expression used to compute the Laplace transform $F(s)$ from an original function $f(t)$ or explain why it doesn't exist.

$$F(s) = \int_0^\infty e^{-st} f(t)\,dt$$

2. (*2 points.*) If such a formula exists, write down a general expression used to compute the INVERSE Laplace transform $f(t)$ from an original function $F(s)$ or explain why it doesn't exist.

No explicit formula for $\mathcal{L}^{-1}[F(s)] = f(t)$

Can't invert this definite integral explicitly

3. (*2 points.*) Explain in your own words, the process by which one uses Laplace Transforms to solve an ordinary differential equation of the form $y'' + py' + qy = f(t)$ with $y(0) = a$ and $y'(0) = b$ for the solution $y(t)$. [I'm looking for sentences describing a multi-step algorithm, no actual calculations!]

$\mathcal{L}[y''+py'+y] = \mathcal{L}[f]$
$s^2Y - sy(0) - y'(0)$
$psY - py(0) + Y = F$

① Apply Laplace Transforms to each element
② Solve for $Y(s)$
③ Invert $Y(s)$ to produce $y(t)$

4. (*2 points.*) Explain the relationship between the Diract Delta Function $\delta(t-a)$ and the Heaviside Function $\mathcal{H}(t-a)$ and explain what kind of actual phenomena (for example, specific input into a harmonic oscillator) each of these special Functions can be used to represent.

$$\int_0^x \delta(t-a)\,dt = \mathcal{H}(x)$$

or $\delta(t-a) = \dfrac{d}{dt}\mathcal{H}(t-a)$

δ : represents a sudden impulse (large input over small time)

$\mathcal{H}$: an input turning on or off suddenly

5. (*2 points.*) The convolution theorem is $\mathcal{L}\left[\int_0^t f(\tau)g(t-\tau)\,d\tau\right] = F(s)G(s)$. Pick any of your two different favorite (non-zero) functions and confirm this result. (HINT: use "easy" functions from the table on reverse!)

$f(t) = 1$
$g(t) = 2$

$$\mathcal{L}\left[\int_0^t 1\cdot 2\,dt\right] = \mathcal{L}[2t] = \frac{2}{s^2} = \frac{1}{s}\cdot\frac{2}{s} \checkmark$$

BONUS (*2 points.*) Provide some thoughts about the course in general. What was the most interesting topic in the course? What was the topic you understood and enjoyed the most? What was the topic you enjoyed the least?