Your Name: BUCKMIRE **SCORE:** /10

INSTRUCTIONS: Answer the following short-answer questions (in 10 minutes).
GOAL: This reading quiz is designed to illuminate your understanding of the concepts in Chapter 3 of the book: General Solution to a 2-D Linear Systems, Phase Planes for Linear Systems with Real Eigenvalues, Complex Eigenvalues, A Zero Eignevalue and Repeated Eigenvalues, Second-Order Linear Equations.

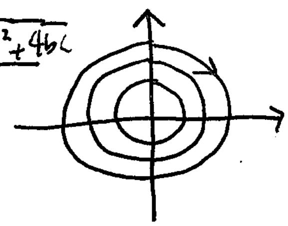
1. (*10 points.*) Write down examples of five differential equations of the form $\frac{d\vec{x}}{dt} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \vec{x}$ (where a, b, c and d are real numbers) that have qualitatively different phase portraits from each other. Describe the equilibrium point and provide a (very rough) little sketch (with arrows!) of the phase portrait of each of your linear systems of ODEs. For each example, explain how you know how (i.e. what information you use) to classify your given examples in the way that you have.

(A) $\lambda = \pm Bi$

$\lambda^2 - (a+d)\lambda + ad - bc = 0$ $\lambda = \frac{(a+d) \pm \sqrt{(a-d)^2 + 4bc}}{2}$

$(a-d)^2 + 4bc < 0$

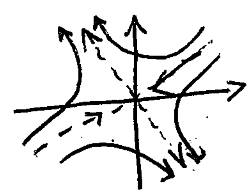
choose $\begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix} \Rightarrow$



CENTER

(B) $\lambda = 1, -1$

$\lambda^2 - 1 = 0$ choose $\begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$

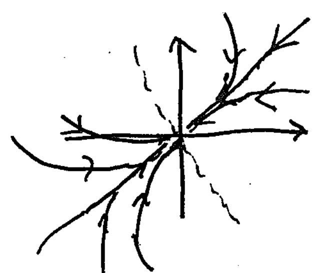


SADDLE

(C) $\lambda = -2, -1$

$(\lambda - 2)(\lambda - 1) = \lambda^2 + 2\lambda + 2$

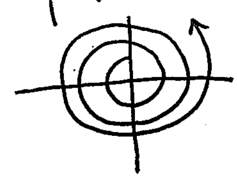
choose $\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$



SINK

(D) $\lambda = \alpha \pm \beta i, \alpha > 0$

Choose $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

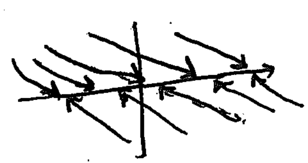


SPIRAL SOURCE

(E) $\lambda = 0, -2$

$\lambda(\lambda - 2) = 0$

$\lambda^2 - 2\lambda = 0$ Choose $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$



ZERO EIG

BONUS (*2 points.*) Provide some thoughts about the course project. What are your biggest apprehensions (fears) or concerns about the project? What information or assistance could I provide you in helping you to address these concerns?

Recall diagonal matrices have eigenvalues along diagonal

i.e. $A = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$