MATH 341: DIFFERENTIAL EQUATIONS — Reading Quiz **TWO** (Sections 1.5-1.9 of Blanchard)

September 22, 2008 — *Prof. R. Buckmire*

Your Name: BUCKMIRE **SCORE:** /10

INSTRUCTIONS: Answer the following short-answer questions (in 10 minutes).
GOAL: This reading quiz is designed to illuminate your understanding of the concepts in the second half of the first Chapter of the book: Existence and Uniqueness Theorem, Phase Lines, Equilibria, Bifurcations, Linear Equations, and Integrating Factors.

1. (*2 points.*) In order for a solution $y(t)$ to $y' = f(t, y)$, $y(t_0) = y_0$ to exist AND be unique for all t what conditions must hold? Provide an example of an IVP which has these properties, if you think it is possible.

If $f(t,y)$ and $\frac{\partial f}{\partial y}(t,y)$ would be everywhere continuous $\Rightarrow$ $y(t)$ exist & unique everywhere

e.g. $y' = y,\ y(0) = 1$

2. (*2 points.*) In your own words, explain what happens at a bifurcation value.

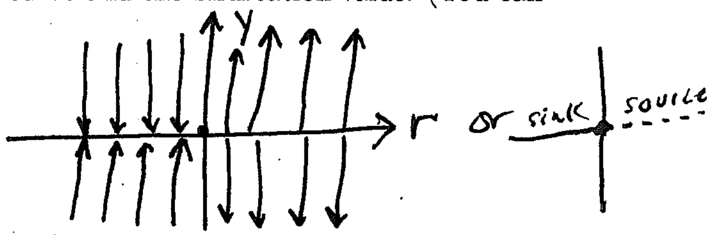
A qualitative change in the fixed points occurs at this value of a parameter.

3. (*2 points.*) Draw the bifurcation diagram for the following DE $y' = ry$ where r is a parameter. Clearly indicate the stable and unstable sections of the curve and the bifurcation value. (You can use the back of this sheet)

bifurcation value

$f(y;r) = ry = 0$

$f'(y) = \boxed{r = 0}$



4. (*2 points.*) TRUE or FALSE: "If $y(t)$ is a solution of a non-homogeneous, linear, ordinary differential equation, then $cy(t)$ (where c is any real number) must also be a solution." PROVE YOUR ANSWER or provide a counter-example.

FALSE!

$Ly = f$

$f \stackrel{?}{=} L(cy) = c(Ly) = cf \neq f$ unless $c = 1$.

5. (*2 points.*) What is the integrating factor $\mu(x)$ that you would have to multiply the DE $y' = -3y\ln(x) + 1$ by in order to obtain a general solution? [DO NOT SIMPLIFY YOUR ANSWER!]

$\mu(x) = e^{\int 3\ln(x)\,dx}$

Given $y' + 3y\ln x = 1$ or $y' + P(x)y = Q(x)$ $\mu(x) = e^{\int P(x)\,dx}$

BONUS (*2 points.*) Comment on the pace of the class. Is it going too fast, too slow or "just right"? What is the concept you think that you least understand in these sections of the text?