Quiz **7** Differential Equations

Name: ______________________________

Wednesday November 12
Ron Buckmire

Time Begun: ____________________
Time Ended: ____________________

Topic : Laplace Transforms

The idea behind this quiz is to provide you with an opportunity to demonstrate your comfort with Lapace Transofrms and be introduced to a special function.

Reality Check:

EXPECTED SCORE : __________**/10** ACTUAL SCORE : __________**/10**

Instructions:

0. Please look for a hint on this quiz posted to `faculty.oxy.edu/ron/math/341/08/`
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday November 17**, at the beginning of class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. The **Gamma Function** is defined as

$$\Gamma(\alpha) = \int_0^\infty e^{-t} t^{\alpha-1}\, dt, \quad (\alpha > 0).$$

(a) *1 point.* Show that $\Gamma(1) = 1$.

(b) *2 points.* Show that $\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$.

(c) *3 points.* Use your results in (a) and (b) to show that $\Gamma(n + 1) = n!$, where n is a positive integer. (HINT: use mathematical induction).

(d) *4 points.* Use all the previous results to show that $\mathcal{L}[t^\alpha] = \dfrac{\Gamma(\alpha+1)}{s^{\alpha+1}}$ and $\mathcal{L}[t^n] = \dfrac{n!}{s^{n+1}}$ when n is a positive integer.