Quiz **6** DIFFERENTIAL EQUATIONS

Name: ______________________________

Friday October 31
Ron Buckmire

Time Begun: ____________________
Time Ended: ____________________

Topic : Bifurcations in Quasi-Linear Systems of Differential Equations

The idea behind this quiz is to provide you with an opportunity to think about how bifurcations can occur in non-linear systems of DEs and practice the analytical technique of Linearization.

Reality Check:

EXPECTED SCORE : __________**/10** ACTUAL SCORE : __________**/10**

Instructions:

0. Please look for a hint on this quiz posted to `faculty.oxy.edu/ron/math/341/08/`
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday November 3**, at the beginning of class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. Consider the linear system of ordinary differential equations with a parameter α

$$\begin{aligned} \frac{dx}{dt} &= y + \alpha x(x^2 + y^2) \\ \frac{dy}{dt} &= -x + \alpha y(x^2 + y^2) \end{aligned}$$

(a) *2 points.* Use linearization to show that regardless of the value of α, the equilibrium at $(0,0)$ is a center.

(b) *2 points.* Suppose $\alpha \neq 0$. Let $r^2 = x^2 + y^2$. Use the fact that $r\frac{dr}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$ and the given nonlinear system of ODEs to produce the expression $\frac{dr}{dt} = \alpha r^3$.

(c) *3 points.* Suppose $\alpha < 0$ (say $\alpha = -1$). Using separation of variables, integrate the differential equation $\frac{dr}{dt} = \alpha r^3$ to obtain an expression for $r(t)$ and show that as $t \to \infty$, $r \to 0$ which means that the center you found in part (a) is *asymptotically* **stable**.

(d) *3 points.* Suppose $\alpha > 0$ (say $\alpha = +1$). Using separation of variables, integrate the differential equation $\frac{dr}{dt} = \alpha r^3$ to obtain an expression for $r(t)$ and show that as t increases, $r \to +\infty$ which means that the center you found in part (a) is **unstable**.