Math 341 Fall 2008 **SHOW ALL YOUR WORK** Quiz **Four**

1. Consider the system of ordinary differential equations

$$\frac{d\vec{x}}{dt} = A\vec{x} = \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix} \vec{x} \text{ where } \vec{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

(a) *4 points.* Show that the matrix A has eigenvalues 0 and -1 and eigenvectors which are multiples of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$. Write down the general solution of the system.

$\begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\qquad \vec{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \lambda = 0$

$\begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} = -1\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ $\qquad \vec{x} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \quad \lambda = -1$

$\vec{x} = c_1 e^{-t}\begin{pmatrix} -2 \\ 1 \end{pmatrix} + c_2 e^{0}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = c_1 e^{-t}\begin{pmatrix} -2 \\ 1 \end{pmatrix} + c_2\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

(b) *3 points.* Find the exact solution for each of the trajectories which go through the points (1, 1),(0, −2) and (4, 0).

$t=0, \begin{pmatrix} 1 \\ 1 \end{pmatrix} = c_1\begin{pmatrix} -2 \\ 1 \end{pmatrix} + c_2\begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow c_1 = 1, c_2 = 3$

$\vec{x}(t) = \begin{pmatrix} -2e^{-t}+3 \\ e^{-t} \end{pmatrix}$

$t=0, \begin{pmatrix} 0 \\ -2 \end{pmatrix} = c_1\begin{pmatrix} -2 \\ 1 \end{pmatrix} + c_2\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad c_1 = -2, c_2 - 2c_1 = 0 \Rightarrow c_2 = 2c_1 = -4$

$\vec{x}(t) = -2e^{-t}\begin{pmatrix} -2 \\ 1 \end{pmatrix} - 4\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4e^{-t}-4 \\ -2e^{-t} \end{pmatrix}$

$t=0, \begin{pmatrix} 4 \\ 0 \end{pmatrix} = c_1\begin{pmatrix} -2 \\ 1 \end{pmatrix} + c_2\begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow c_1 = 0, c_2 = 4$

$\vec{x}(t) = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

(c) *3 points.* On the figure below clearly indicate where each of the trajectories of the solutions which start at (1, 1), (0, −2) and (4, 0) ends up as $t \to \infty$.