Math 341 Fall 2008

Quiz **3** **Differential Equations**

Name: _____

Friday September 26

Time Begun: _____ Ron Buckmire

Time Ended: _____

Topic : Systems of First Order ODEs (Lotka-Volterra)

The idea behind this quiz is to provide you with an opportunity to illustrate your ability to combine your knowledge of systems of ODEs with prior knowledge.

Reality Check:

EXPECTED SCORE : _____**/10** ACTUAL SCORE : _____**/10**

Instructions:

0. Please look for a hint on this quiz posted to `faculty.oxy.edu/ron/math/341/08/`
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due on Monday September 29**, at the beginning of class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Math 341 Fall 2008 **SHOW ALL YOUR WORK** Quiz **Three**

1. Consider the standard Lotka-Volterra predator-prey system of ODEs

$$\frac{dR}{dt} = \alpha R - \beta RF, \qquad \frac{dF}{dt} = -\gamma F + \delta RF$$

where a, b, c, e are positive parameters and $R(t)$ and $F(t)$ represents poopulations of rabbits and foxes, respectively.

(a) *4 points.* What are the R-nullclines and F-nullclines for this system? (Sketch the curves in the RF-plane along which $R' = 0$ or $F' = 0$ in the space below.)

(b) *1 point.* Label your curves in part (a) and explain what the significance of any intersections of the nullclines are.

(c) *1 point.* Assuming that R' is never zero or undefined for a particular time interval show that $\dfrac{dF}{dR} = \dfrac{F(\delta R - \gamma)}{R(\alpha - \beta F)}$.

(d) *3 points.* Since the equation in **(c)** is separable, show that one can solve it to obtain a family of implicitly defined solution curves given by the equation below (where K is an unknown constant)

$$F^{\alpha} R^{\gamma} e^{-(\beta F + \delta R)} = K.$$

(e) *1 point.* Sketch the solution curves for at least two different values of K in the RF-plane given in part (a).