Math 341 Fall 2008 **SHOW ALL YOUR WORK** Quiz Three

1. Consider the standard Lotka-Volterra predator-prey system of ODEs

$$\frac{dR}{dt} = \alpha R - \beta RF, \qquad \frac{dF}{dt} = -\gamma F + \delta RF$$

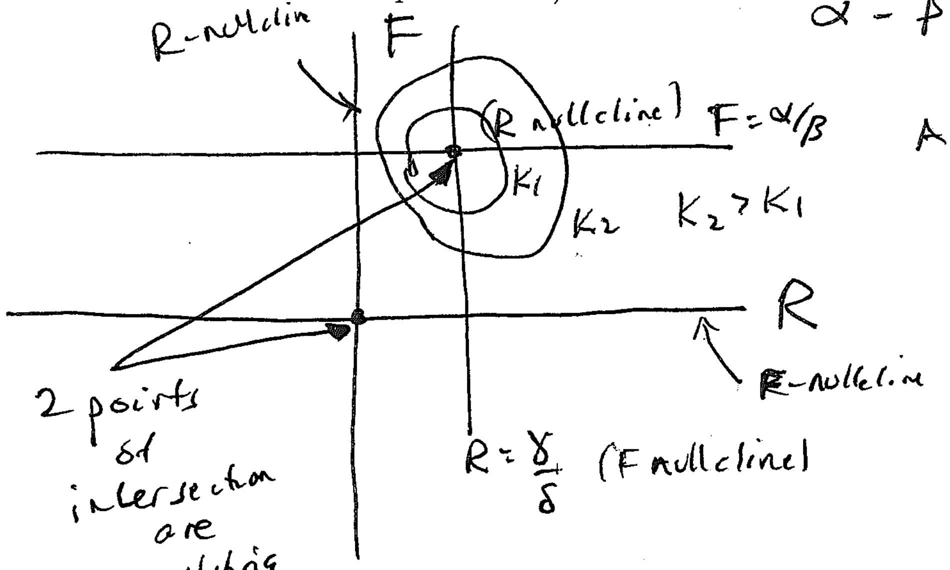
where a, b, c, e are positive parameters and $R(t)$ and $F(t)$ represents popqulations of rabbits and foxes, respectively.

(a) *4 points.* What are the R-nullclines and F-nullclines for this system? (Sketch the curves in the RF-plane along which $R' = 0$ or $F' = 0$ in the space below.)

$\alpha - \beta F = 0 \Rightarrow F = \frac{\alpha}{\beta}$ or $R = 0$ AND $F = 0$ or $\delta R - \gamma = 0$, $R = \frac{\gamma}{\delta}$

F-nullclines $F = 0$ and $R = \gamma/\delta$

R-nullclines $R = 0$ and $F = \alpha/\beta$



(b) *1 point.* Label your curves in part (a) and explain what the significance of any intersections of the nullclines are.

Intersections of nullclines represent equilibria locations

(c) *1 point.* Assuming that R' is never zero or undefined for a particular time interval show that $\frac{dF}{dR} = \frac{F(\delta R - \gamma)}{R(\alpha - \beta F)}$.

$$\frac{dF}{dR} = \frac{F'}{R'} = \frac{-\gamma F + \delta RF}{\alpha R - \beta RF} = \frac{F(\delta R - \gamma)}{R(\alpha - \beta F)}$$

(d) *3 points.* Since the equation in **(c)** is separable, show that one can solve it to obtain a family of implicitly defined solution curves given by the equation below (where K is an unknown constant)

$$F^{\alpha} R^{\gamma} e^{-(\beta F + \delta R)} = K.$$

$$\alpha - \beta F \frac{dF}{F} = dR \frac{\delta R - \gamma}{R}$$

$$\int \frac{\alpha}{F} - \beta \, dF = \int \delta - \frac{\gamma}{R} \, dR \Rightarrow \alpha \ln F - \beta F = \delta R - \gamma \ln R + C$$

$$\alpha \ln F - \beta F - \delta R + \gamma \ln R = C$$

(e) *1 point.* Sketch the solution curves for at least two different values of K in the RF-plane given in part (a).

$$\ln F^{\alpha} + \ln(e^{-\beta F}) + \ln(e^{-\delta R}) + \ln R^{\gamma} = C$$

$$\ln(F^{\alpha} R^{\gamma} e^{-\beta F} e^{-\delta R}) = C$$

$$F^{\alpha} R^{\gamma} e^{-(\beta F + \delta R)} = e^{C} = K$$