1. Consider the standard Lotka-Volterra predator-prey system of ODEs

\[ \frac{dR}{dt} = \alpha R - \beta RF, \quad \frac{dF}{dt} = -\gamma F + \delta RF \]

where \(a, b, c, \gamma \) are positive parameters and \( R(t) \) and \( F(t) \) represent populations of rabbits and foxes, respectively.

(a) 4 points. What are the \( R \)-nullclines and \( F \)-nullclines for this system? (Sketch the curves in the \( RF \)-plane along which \( R' = 0 \) or \( F' = 0 \) in the space below.)

(b) 1 point. Label your curves in part (a) and explain what the significance of any intersections of the nullclines are.

(c) 1 point. Assuming that \( R' \) is never zero or undefined for a particular time interval show that

\[ \frac{dF}{dR} = \frac{F(\delta R - \gamma)}{R(\alpha - \beta F)} \]

\[ \frac{dF}{dR} = \frac{F'}{R'} = \frac{\delta F + \delta RF}{\alpha R - \beta RF} = \frac{F(\delta R - \gamma)}{R(\alpha - \beta F)} \]

(d) 3 points. Since the equation in (c) is separable, show that one can solve it to obtain a family of implicitly defined solution curves given by the equation below (where \( K \) is an unknown constant)

\[ F^\alpha R^\gamma e^{-(\beta F + \delta R)} = K. \]

\[ \frac{\alpha - \beta F}{F} \frac{dF}{dR} = \alpha R \frac{\delta R - \gamma}{R} \]

\[ \int \frac{\alpha - \beta F}{F} dF = \int \frac{\delta R - \gamma}{R} dR \quad \Rightarrow \quad \alpha \ln F - \beta F = \delta R - \gamma \ln R + C \]

\[ \alpha \ln F - \beta F - \delta R + \gamma \ln R = C \]

(e) 1 point. Sketch the solution curves for at least two different values of \( K \) in the \( RF \)-plane given in part (a).

\[ \ln \left( F^\alpha R^\gamma e^{-(\beta F + \delta R)} \right) = C \quad \ln \left( F^\alpha R^\gamma \frac{e^{-(\beta F)}}{e^{-(\delta R)}} \right) \]

\[ F^\alpha R^\gamma e^{-(\beta F + \delta R)} = e^C = K \quad \ln F^\alpha + \ln (e^{-\beta F}) + \ln (e^{-\delta R}) \]

\[ \ln = C \quad \ln R^\gamma = C \]