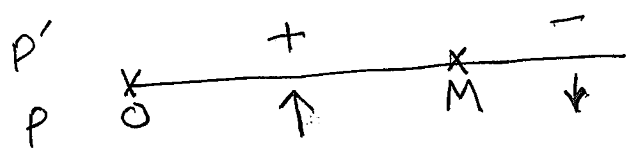
1. Consider the autonomous differential equation $\frac{dP}{dt} = kP(M-P)$ where $P(t)$ is the population in an environment which can only sustain M individuals and k is the constant of proportionality.

(a) *4 points.* Find and classify all the equilibrium points of this differential equation and draw the phase line.

$P' = KP(M-P) = 0 \Rightarrow P = 0$ and $P = M$



$P = M$ is a stable equilibrium point, i.e a sink

$P = 0$ is an unstable equilibrium, i.e. a source

(b) *3 points.* Show that $\frac{d^2P}{dt^2} = 2k^2P(P-\frac{M}{2})(P-M)$ and that $\begin{cases} P'' > 0, & \text{when } 0 < P < \frac{1}{2}M \\ P'' < 0, & \text{when } \frac{1}{2}M < P < M \\ P'' > 0, & \text{when } P > M \end{cases}$

$$\begin{aligned} P'' &= K(M-P)P' + KP(-1)P' \\ &= KP'[M-P-P] \\ &= KP'(M-2P) \\ &= KKP(M-P)(M-2P) \\ &= K^2P(P-M)(2P-M) = 2K^2P(P-M)(P-\tfrac{M}{2}) \end{aligned}$$

(c) *3 points.* Use information from part (a) and part (b) to **carefully** sketch solution curves which go through (i) $P(0) = 2M$, (ii) $P(0) = M/2$ and (iii) $P(0) = M/4$in the space below.