Math 341 Fall 2008 **SHOW ALL YOUR WORK** Quiz One

1. Consider the following differential equation

$$\frac{dy}{dx} = \left(\frac{y}{x}\right)^2 + \frac{y}{x}.$$

(a) *1 point.* Fully classify this differential equation by **type**, **order** and **linearity**.

type: ordinary order: first linearity: nonlinear

(b) *2 points.* Show that the given differential equation when thought of as $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ can be transformed using the transformation $u = y/x$ (i.e. $y = ux$) into a *separable* equation of the form $x\frac{du}{dx} = F(u) - u$ where $F(t) = t^2 + t$.

$$y = ux$$

$$\frac{dy}{dx} = u\frac{dx}{dx} + \frac{du}{dx}\cdot x = u + \frac{du}{dx}x = u^2 + u = F(u) \Rightarrow x\frac{du}{dx} = F(u) - u$$

(c) *4 points.* Use the separation of variables technique to show that the *general* solution to the given differential equation has the form $y = \frac{Cx^2}{1 - Cx}$, where C is an unspecified constant.

$$x\frac{du}{dx} = u^2 + u - u = u^2$$

$$\int\frac{du}{u^2} = \int\frac{dx}{x}$$

$$-\frac{1}{u} = \ln x + C$$

$$u = \frac{-1}{\ln x + C}$$

$$\frac{y}{x} = \frac{-1}{\ln x + C} \Rightarrow y = \frac{-x}{\ln x + C}$$

$$x\frac{du}{dx} = u^2 + 2u - u = u^2 + u$$

$$\frac{du}{u^2 + u} = \frac{dx}{x}$$

$$\int\frac{1}{u(u+1)}du = \int\frac{dx}{x} \Rightarrow \int\frac{-1\,du}{u+1} + \int\frac{1}{u}du = \int\frac{dx}{x}$$

$$\frac{A}{u+1} + \frac{B}{u} = \frac{1}{u(u+1)}$$

$$Au + B(u+1) = 1$$

$$B = 1$$

$$A = -1$$

$$\ln\frac{u}{u+1} = \ln x + C$$

$$\frac{u}{u+1} = Ax$$

$$u = Axu + Ax$$

$$u - Axu = Ax$$

$$u(1 - Ax) = Ax$$

$$u = \frac{Ax}{1 - Ax}$$

$$\frac{y}{x} = \frac{Ax}{1 - Ax}$$

$$y = \frac{Ax^2}{1 - Ax}$$

(d) *3 points.* If possible, find each of the *particular* solutions to the differential which go through the points $(1, 1)$, $(1, 0)$ and $(0, 1)$ in the xy-plane, respectively. DISCUSS YOUR ANSWERS.

If $x = 1$, $y = 1$ corresponds to $u = 1$

$$1 = \frac{-1}{\ln(1) + C} \Rightarrow C = -1$$

$$\boxed{y = \frac{-x}{\ln(x) - 1}}$$

If $x = 0$, $y = 1 \Rightarrow u$ is undefined

$x = 0$ NOT in domain of validity of DE, so no particular sol^n.

If $x = 1$, $y = 0$ corresponds to $u = 0$

$$0 = \frac{-1}{\ln(1) + C} = -\frac{1}{C}$$

No value for C, but $\boxed{y = 0}$ is a sol^n

For $(1,1)$ $1 = \frac{A}{1-A} \Rightarrow A = \frac{1}{2}$ $\boxed{y = \frac{x^2}{2-x}}$

For $(1,0)$ $A = 0$ so $\boxed{y = 0}$

For $(0,1)$ $1 = 0$ doesn't make