Math 341 Fall 2008 **SHOW ALL YOUR WORK** **Bonus Quiz FOUR**

1. Consider the following "auto-convolution" equation

$$f * f = \int_0^t f(\tau)f(t-\tau)\, d\tau = 6t^3$$

(a) *5 points.* **Show** that the function $f(t) = \pm 6t$ is the solution of the above equation. (Solve the integral equation using Laplace Transforms and you should find that the solution is indeed the given function.)

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}[f*f] = \mathcal{L}[6t^3]$$

$$[F(s)]^2 = 6\mathcal{L}[t^3]$$

$$= 6 \cdot \frac{3!}{s^4}$$

$$F \cdot F = \frac{6}{s^2} \cdot \frac{6}{s^2}$$

$$\Rightarrow F = \pm \frac{6}{s^2} \Rightarrow f(t) = \mathcal{L}^{-1}\left[\frac{6}{s^2}\right] = 6\mathcal{L}^{-1}\left[\frac{1}{s^2}\right]$$

$$= 6 \cdot \frac{t}{1!} = 6t$$

$$f(t) = \pm 6t$$

(b) *5 points.* **Confirm** that if $f(t) = \pm 6t$, then the convolution of $f(t)$ with itself is $6t^3$. (Check that the given function indeed satisfies the given equation when "auto-convolved.")

$$\int_0^t 6\tau \cdot 6(t-\tau)\,d\tau = \int_0^t 36(t\tau - \tau^2)\,d\tau = 36\int_0^t t\tau - \tau^2\,d\tau$$

$$= 36\left(t\frac{\tau^2}{2} - \frac{\tau^3}{3}\right)\Big|_0^t = 36\left[\frac{t^3}{2} - \frac{t^3}{3}\right] = 36\frac{t^3}{6} = 6t^3 \checkmark$$