Math 341 Fall 2008 **SHOW ALL YOUR WORK** **Bonus Quiz TWO**

1. Consider the following one-parameter family of nonlinear first-order differential equations where α is a known real parameter value

$$\frac{dy}{dx} = y^2 - \alpha y + 1.$$

(a) *1 point.* Show that this DE has no equilibrium points for $|\alpha| < 2$.

$f(y;\alpha) = y^2 - \alpha y + 1 = 0 \Rightarrow y = \dfrac{\alpha \pm \sqrt{(-\alpha)^2 - 4\cdot 1}}{2\cdot 1} = \dfrac{\alpha \pm \sqrt{\alpha^2-4}}{2}$

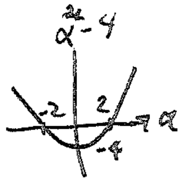
$f'(y;\alpha) = 2y - \alpha = 0$

$y = \dfrac{\alpha}{2}$

For real answers $\alpha^2 - 4 \geq 0 \Rightarrow \alpha^2 \geq 4 \Rightarrow$

So, bifurcation value is $\alpha^2 - 4 = 0 \Rightarrow \alpha = \pm 2$.

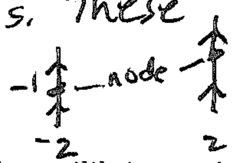
$\alpha^2 - 4$



$\alpha \leq -2$ and $\alpha \geq 2$ so $|\alpha| \geq 2$. When $|\alpha| < 2$ no equil'm pts

(b) *1 point.* For what values of α will the DE have exactly one equilibrium point? Classify the equilibrium point in this case and give the constant solution.

If $\alpha^2 - 4 = 0$, $y = \frac{\alpha}{2}$ will be the one equilibrium point, in this case there are two values for α, $\alpha = \pm 2$ so $y = \pm 1$ will be equilibrium solutions. These ~~solution~~ values will be nodes.



(c) *2 points.* Show that when $|\alpha| > 2$ the DE has exactly one stable equilibrium point (sink) and one unstable equilibrium point (source). Give all the constant solutions.

If $|\alpha| > 2 \Rightarrow \alpha^2 > 4 \Rightarrow \alpha^2 - 4 > 0$

For these values

$y = \dfrac{\alpha \pm \sqrt{\alpha^2-4}}{2} = \dfrac{\alpha}{2} \pm \sqrt{\dfrac{\alpha^2}{4} - \dfrac{4}{4}} = \dfrac{\alpha}{2} \pm \sqrt{\left(\dfrac{\alpha}{2}\right)^2 - 1}$

$f'(y;\alpha) = 2\left[\dfrac{\alpha}{2} \pm \sqrt{\left(\dfrac{\alpha}{2}\right)^2 - 1}\right] - \alpha = \pm 2\sqrt{\left(\dfrac{\alpha}{2}\right)^2 - 1} = \pm\sqrt{\alpha^2 - 4}$

When $|\alpha| > 2$

$y = \sqrt{\alpha^2 - 4}$ is a source (unstable equilibrium) since $f'(y;\alpha) > 0$

$y = -\sqrt{\alpha^2 - 4}$ is a sink (stable equilibrium) since $f'(y;\alpha) < 0$

(d) *1 point.* Use your answers from above to sketch the bifurcation diagram for the given DE. (HINT: think about what happens to equilibrium solutions as $\alpha \to \pm\infty$!)