BONUS QUIZ 1

Differential Equations

Name: ______________________________

Wednesday September 10
Prof. Ron Buckmire

Topic : Analyzing a Clairault Equation

The idea behind this bonus quiz is to provide you with an opportunity to illustrate your understanding of singular solutions to ordinary differential equations.

Reality Check:

EXPECTED SCORE : __________**/5**   ACTUAL SCORE : __________**/5**

Instructions:

0. Please look for a hint on this quiz posted to `faculty.oxy.edu/ron/math/341/08/`
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This bonus quiz is due on Friday September 12**, with the rest of your week's homework. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. Consider the first-order, nonlinear, Clairault ordinary differential equation

$$y = x\left(\frac{dy}{dx}\right) - \frac{1}{4}\left(\frac{dy}{dx}\right)^2$$

(a) *1 point.* Confirm that the family of solutions is the set of **lines** $y = Cx - \frac{1}{4}C^2$.

(b) *3 points.* Show that the lines $y = Cx - \frac{1}{4}C^2$ are tangent to the curve $y = x^2$ at the point $\left(\frac{C}{2}, \frac{C^2}{4}\right)$ and sketch the curve and its tangents below for at least 4 values of C.

(c) *1 point.* Explain how parts (a) and (b) imply that $y = x^2$ is a singular solution of the given Clairault equation.