Math 341 Spring 2005 **SHOW ALL YOUR WORK** **Bonus Quiz One**

1. Consider the first-order, nonlinear Clairault equation

$$y = x\left(\frac{dy}{dx}\right) - \frac{1}{4}\left(\frac{dy}{dx}\right)^2$$

(a) *3 points.* Confirm that the family of solutions is the set of **lines** $y = Cx - \frac{1}{4}C^2$.

$\frac{dy}{dx} = C$ when $y = x \cdot C - \frac{1}{4}C^2$, thus it solves the DE

(b) *5 points.* Show that the lines $y = Cx - \frac{1}{4}C^2$ are tangent to the curve $y = x^2$ at the point $\left(\frac{C}{2}, \frac{C^2}{4}\right)$ and sketch the curve and its tangents below for at least 4 values of C.

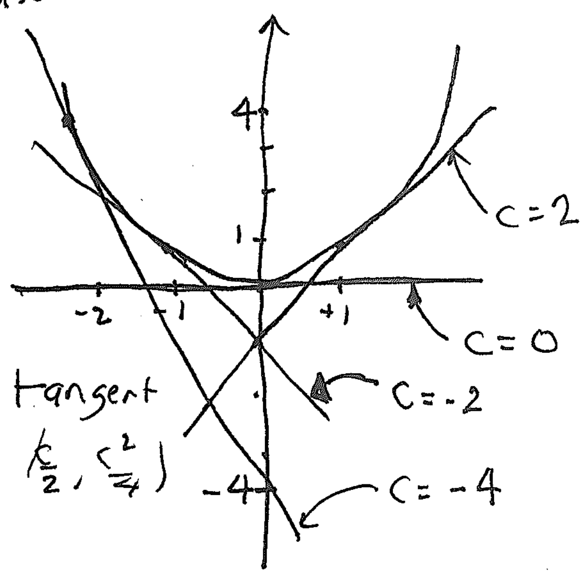
$\frac{dy}{dx} = 2x$ at $x = \frac{C}{2} \Rightarrow \frac{dy}{dx} = 2 \cdot \frac{C}{2} = C$

$y - \frac{C^2}{4} = C \cdot \left(x - \frac{C}{2}\right)$

$y - \frac{C^2}{4} = Cx - \frac{C^2}{2}$

$y = Cx - \frac{C^2}{2} + \frac{C^2}{4}$

$y = Cx - \frac{C^2}{4}$ ← eqⁿ of tangent line at $\left(\frac{C}{2}, \frac{C^2}{4}\right)$



(c) *2 points.* Explain how parts (a) and (b) imply that $y = x^2$ is a singular solution of the given Clairault equation.

$y = x^2$ is a solution of the Clairault Equation

$\frac{dy}{dx} = 2x \Longrightarrow y = x^2 = x \cdot (2x) - \frac{1}{4}(2x)^2$

$= x \cdot 2x - \frac{1}{4} 4x^2$

$= 2x^2 - x^2$

$= x^2$ ✓

But $y = x^2$ is not in the family of solutions, it is tangential to them (this is what we expect singular solutions to do)