

This handout is modified from an original written by **Professor Angela Gallegos** in Fall 2006.

To continue our discussion of visualizing phase planes and phase portraits, you are first going to visualize some phase planes using two different technological sources. You will be working with the following systems and you will have different questions to answer using each tool.

System A. $\frac{dx}{dt} = y; \frac{dy}{dt} = -x.$

System B. $\frac{dx}{dt} = 5x; \frac{dy}{dt} = -y.$

System C. $\frac{dx}{dt} = x + y; \frac{dy}{dt} = 4x - 2y.$

System D. $\frac{dx}{dt} = y; \frac{dy}{dt} = -\frac{k}{m}x.$

Using your text cd...

To use the text cd program from the lab computers, look under:

Start--> My Computer-->S ('stuserver')--> Math Courses--> Math 341-->Fall 2008.

Then you can double click on

`Differential Equations.exe`

Once you open that program, a window should pop up with lots of program names listed. You are interested in the one called `HPGSystemSolver`. Click on it. Again, a window should pop up which allows you to enter the differential equations you are interested in over your variable range of interest. When you are authoring a differential equation expression, a guide will pop up that shows you the syntax for several different mathematical expressions. Again the \wedge is used for exponentiation and the $*$ is used for multiplication. After you have entered both differential equations, click on **Show Field** and the direction field should appear. The default settings will allow you to generate solutions by clicking on a point in the direction field. Notice that when you do that, the solution trajectory in the phase plane will be traced out for each variable versus time.

While you are in the `HPGSystemSolver` use the advantage of the different settings to answer the following questions. When you are considering the last system, just pick some values for k/m (Try some less than one and some bigger than one. Note: $k = m$ is System A.)

1. From looking at the direction field, where do you expect an equilibrium solution to be?
2. What do you notice about the time scale of solutions when you look at the solutions versus time plots? How does this compare to the time behavior you expect from simply looking at the trajectories in the phase plane?
3. How does the solution behavior differ between the phase planes for each of the systems?

Using Matlab...

John Polking at Rice University has created a Matlab code that generates slope fields for differential equations. You can gain access to this code by typing

`pplane`

into Google. “ODE Software for MATLAB” should be the first link that pops up. Click on it. (Alternatively, you can directly head to the **Resources** page of our course website and download the file `pplane77.m` and save the file to your computer. Please be aware of the location you saved it to.

Now you can open Matlab on your computer. It is under

`Start--> All Programs--> Mathematics --> MATLAB.`

Once you have the Matlab window open, you should see the three part GUI window. Like before, change the **Current Directory** to the same directory where you saved the file `pplane7`. At the prompt in the command window type

`pplane7`

and hit return. A GUI window should open up in which you can type the differential equations you are interested in and set the variable range over which you can visualize the phase plane. In the default settings, the direction field appears, but notice that you can change it. When you have entered the expression, click on **proceed** and your slope field should pop up. You can click on any point in the slope field window and a solution trajectory should be drawn. (You can erase the solutions by going to **Edit --> Erase all solutions.**) One advantage the `pplane7` has is that you can enter System D in terms of k and m . Then you can set values for k and m in the parameter values boxes and change them there. Do this.

Because you have visualized the phase plane before, the solution behavior should not come as a surprise to you. But you should notice if there are features in either program that lend themselves towards answering certain questions.

Besides entering the parameters symbolically, there is one other feature of the Matlab program that I want you to use. Input any system so that the vector field is displayed. Then, in the display window, go to **Solutions --> Show Nullclines**. Some lines should appear. Look at the lines for each system and **answer the following questions**:

1. What is the equation of each of the lines?
2. Where, if anywhere, do the lines intersect?
3. How do the equations of the lines and the point of intersection (if there is one) relate to the system of equations?