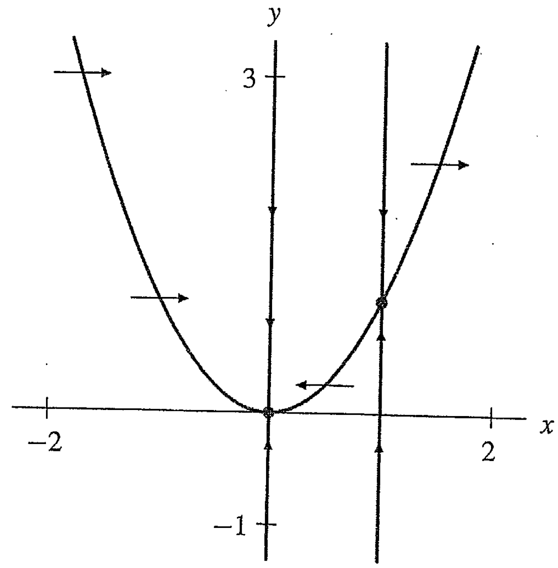
3. For the x-nullcline, $x(x-1)=0$, or $x=0$ and $x=1$, and for the y-nullcline, $y=x^2$. The equilibrium points are the intersection points of the x- and y-nullclines. They are $(0,0)$ and $(1,1)$.



The initial conditions (a) and (b) are in right-up and left-up region respectively. Therefore, their solution curves eventually enter the region where $y > x^2$ and $x \leq 1$, and tend toward the equilibrium point at $(0, 0)$. The initial condition (c) is on the x-nullcline $x = 1$, and therefore its solution curve tends to the equilibrium point at $(1, 1)$.

4. **(a)** Equilibria are located where x-nullclines and y-nullclines intersect, so those equilibria with both $x > 0$ and $y > 0$ are located on the intersection of the lines

$$Ax + By = C \quad \text{and} \quad Dx + Ey = F.$$

However, the only way that two lines can intersect at more than one point is if they are really the same line. This happens if

$$A/D = B/E = C/F.$$

(b) To guarantee that there is exactly one equilibrium point at which the species coexist, we can stipulate that the x- and y-intercepts of the x- and y-nullclines are positioned so that these two lines are forced to intersect in the first quadrant. For example, we could require that the y-intercept of the x-nullcline, namely C/B, lies below the y-intercept of the y-nullcline, namely F/E, whereas the opposite happens for the x-intercepts. That is, we could require that

$$F/E > C/B \quad \text{but} \quad F/D < C/A.$$

Reversing both of these inequalities also guarantees that the species can coexist.

15. **(a)** Since the species are cooperative, an increase in y results in an increase in x and vice versa. Therefore, one needs to change the signs in front of B and D from $-$ to $+$.

(b) The x-nullcline is given by $x = 0$ or $-Ax + By + C = 0$. The y-nullcline is given by $y = 0$ or $Dx - Ey + F = 0$. The origin is always an equilibrium point. Also, $x = 0$, $y = F/E$ and $x = C/A$, $y = 0$ are equilibrium points. Equilibrium points with both x and y positive arise from solutions of

$$\begin{cases} -Ax + By + C = 0 \\ Dx - Ey + F = 0 \end{cases}$$

In matrix notation, we obtain

$$\begin{pmatrix} -A & B \\ D & -E \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -C \\ -F \end{pmatrix}.$$

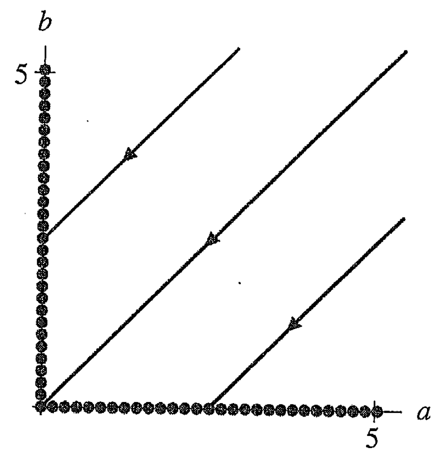
In order for a unique solution to exist, $AE - BD \neq 0$. Then, the solution is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{AE - BD} \begin{pmatrix} CE + BF \\ CD + AF \end{pmatrix}$$

Since A through F are all positive, we must have $AE - BD > 0$ for the solution to be in the first quadrant.

If $AE - BD = 0$, then $-Ax + By$ must be a negative multiple of $Dx - Ey$, so there are no solutions with both x and y positive.

16. **(a)** The a- and b-nullclines are identical. They both consist of both the a- and the b-axis. Hence all points on either of these axes are equilibrium points.



(b) Since $da/dt = db/dt$, solution curves are lines of slope 1 in the ab-plane.

(c) Along these lines, the solutions tend to the equilibrium point that is the intersection point of this line and either the positive a- or b-axis.

17. **(a)** For the a-nullcline, $da/dt = 0$, so $2 - ab/2 = 0$, or $ab = 4$. For the b-nullcline, $db/dt = 0$, so $ab = 3$. Both nullclines are hyperbolas, and the curve of $ab = 4$ is above the one of $ab = 3$. Therefore, the direction of vector field on $ab = 4$ is vertical and downward, and the one on $ab = 3$ is horizontal and to the right.