(b) If the spring provides slightly less force for a given compression or extension, k decreases slightly. Then, γ decreases slightly, the period increases slightly, and the clock runs slow.

(c) The behavior of the period with respect to the mass is more complicated. We compute the derivative of P with respect to m and obtain

$$\frac{dP}{dm} = 4\pi(4mk - b^2)^{-3/2}(2mk - b^2).$$

Hence, the sign of dP/dm is determined by the sign of $(2mk - b^2)$. If $b < \sqrt{2mk}$, then P increases if m increases slightly. If $b > \sqrt{2mk}$, the P decreases if m increases slightly.

(d) It is possible to have the effects cancel out, so any result (fast, slow or unchanged) is possible.

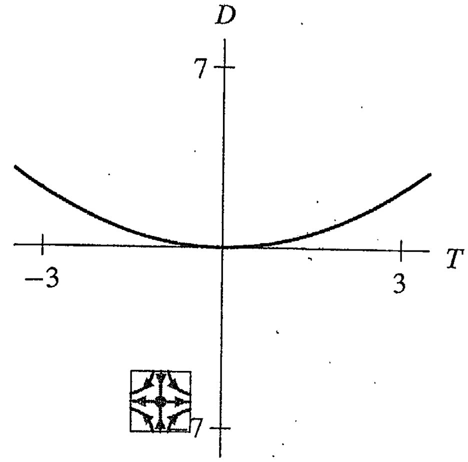
EXERCISES FOR SECTION 3.7

1.

Table 3.2
Possibilities for linear systems

type	condition on λ	examples
sink	$\lambda_1 < \lambda_2 < 0$	Sec. 3.7, Fig. 3.52
saddle	$\lambda_1 < 0 < \lambda_2$	Sec. 3.3, Fig. 3.12–3.14
source	$0 < \lambda_1 < \lambda_2$	Sec. 3.3, Fig. 3.19
spiral sink	$\lambda = \alpha \pm i\beta$, $\alpha < 0$, $\beta \neq 0$	Sec. 3.1, Fig. 3.2 and 3.4
spiral source	$\lambda = \alpha \pm i\beta$, $\alpha > 0$, $\beta \neq 0$	Sec. 3.4, Fig. 3.29–3.30
center	$\lambda_1 = \pm i\beta$, $\beta \neq 0$	Sec. 3.1, Fig. 3.1 and 3.3
		Sec. 3.4, Fig. 3.28
sink	$\lambda_1 = \lambda_2 < 0$	Sec. 3.5, Fig. 3.35–3.36
(special case)	One line of eigenvectors	
source	$0 < \lambda_1 = \lambda_2$	Sec. 3.5, Ex. 2
(special case)	One line of eigenvectors	
sink	$\lambda_1 = \lambda_2 < 0$	Sec. 3.5, Ex. 23
(special case)	Every vector is eigenvector	
source	$0 < \lambda_1 = \lambda_2$	Sec. 3.5, Ex. 23
(special case)	Every vector is eigenvector	
no name	$\lambda_1 < \lambda_2 = 0$	Sec. 3.5, Fig. 3.39–3.40
no name	$0 = \lambda_1 < \lambda_2$	Sec. 3.5, Ex. 19
no name	$\lambda_1 = \lambda_2 = 0$	Sec. 3.5, Ex. 21
	One line of eigenvectors	
no name	$\lambda_1 = \lambda_2 = 0$	entire plane of equilibrium points
	Every vector is an eigenvector	

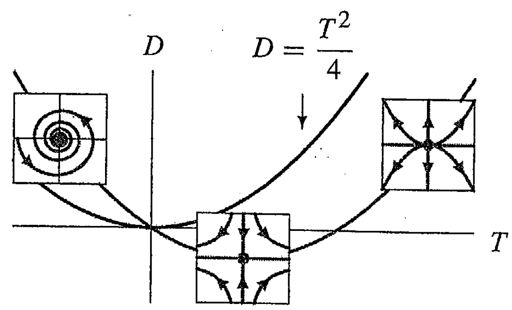
6. **(a)**



(b) The curve in the trace-determinant plane is not a curve at all. For all values of a, $T = -1$ and $D = -6$. So the curve is simply a point in the trace-determinant plane. For all a, we have a saddle.

(c) There are no bifurcations, since the origin is always a saddle. (There is nothing special about $a = 0$, by the way.)

7. **(a)**



(b) The trace T is $2a$, and the determinant D is $a^2 - a$. Therefore, the curve in the trace-determinant plane is

$$\begin{aligned} D &= a^2 - a \\ &= \left(\frac{T}{2}\right)^2 - \frac{T}{2} \\ &= \frac{T^2}{4} - \frac{T}{2}. \end{aligned}$$

This curve is a parabola. It meets the repeated-eigenvalue parabola (the parabola $D = T^2/4$) if

$$\frac{T^2}{4} - \frac{T}{2} = \frac{T^2}{4}.$$

Solving this equation yields $T = 0$, which corresponds to $a = 0$.

This curve also meets the T-axis (the line $D = 0$) if

$$\frac{T^2}{4} - \frac{T}{2} = 0,$$

so if $T = 0$ or $T = 2$, then $D = 0$.