Differential Equations

Math 340 §2 Fall 2015 MWF 3:00-3:55pm Fowler 307

 http://sites.oxy.edu/ron/math/340/15/

Worksheet 13

TITLE Euler's Method for Systems of ODEs
CURRENT READING Blanchard, 2.5

Homework Set #6 (Part 1) due Friday October 2
Section 2.1: 1, 2, 3, 5, 7, 10, 14*. **Section 2.2**: 7, 8, 11, 21* (EXPLAIN!), 24, 26.

Homework Set #6 (Part 2) due Monday October 5
Section 2.4: 2, 5, 7, 8. **Section 2.5**: 2, 3.
Chapter 2 Review: 2, 3, 7, 12, 13 15, 16, 20, 30*.

SUMMARY
It's baaack! We'll look at how to use Euler's Method for estimating solutions to systems of ODEs, i.e. $\frac{d\vec{x}}{dt} = \vec{F}(\vec{x})$.

1. Euler's Method for Systems

The algorithm for generating approximate solutions to the ODE $\frac{d\vec{x}}{dt} = \vec{F}(\vec{x})$ with initial condition $\vec{x}(0) = \vec{x_0}$ is

$$\vec{x}_{new} = \vec{x}_{old} + \vec{F}(\vec{x}_{old})\Delta t$$

EXAMPLE

A lot of the time the systems we will be looking at are systems of two ODEs, so in the case the IVP looks like

$$\begin{aligned} \frac{dx}{dt} &= f(x,y), \qquad x(0) = x_0 \\ \frac{dy}{dt} &= g(x,y), \qquad y(0) = y_0 \end{aligned}$$

The Euler's Method algorithm for a system of two ODEs looks like

$$\begin{aligned} x_{new} &= x_{old} + f(x_{old}, y_{old})\Delta t \\ y_{new} &= y_{old} + g(x_{old}, y_{old})\Delta t \end{aligned}$$

Exercise

Conside the system $\frac{dx}{dt} = x + y$; $\frac{dy}{dt} = 4x - 2y$. Starting at $(x, y) = (1, 0)$ and $\Delta t = 0.5$ let's take two "Euler steps" to approximate the solution curve through this point.

In *Worksheet #10* we were introduced to the Lotka-Volterra model of predator-prey populations.

$$\frac{dR}{dt} = 2R - 1.2RF$$
$$\frac{dF}{dt} = -F + 0.9RF$$

GROUPWORK

Let's use Euler's Method with a $\Delta t = 1$ and the table below to estimate the population of rabbits and foxes after 3 time-steps, starting with $R(0) = 1$, $F(0) = 1$

t	R	F	R'	F'	ΔR	ΔF	Δt

Clearly, the most efficient way to do this would be to use a computer. Go to the computers and look at the spreadsheet `PredatorPrey.xls` on the S-drive and verify (and extend) your calculations.