# Differential Equations

Math 340 §2 Fall 2015 MWF 3:00-3:55pm Fowler 307
 http://sites.oxy.edu/ron/math/340/15/

## *Worksheet 10*

**TITLE** First Order Systems of ODEs
**CURRENT READING** Blanchard, 2.1

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**Homework Set #6 due Friday October 2**
**Section 2.1**: 1, 2, 3, 5, 7, 10, 14*. **Section 2.2**: 7, 8, 11, 21* (EXPLAIN!), 24, 26.
**Section 2.4**: 2, 5, 7, 8. **Section 2.5**: 2, 3.
**Chapter 2 Review**: 2, 3, 7, 12, 13 15, 16, 20, 30*.

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**SUMMARY**
We will be introduced to models that involve a system of first order differential equations. The most famous of these is the Predator-Prey model of Lotka and Volterra.

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### 1. Systems of $n$ First-Order Differential Equations as $n$-th Order DEs

The general form for a system of first order DEs is:

$$\frac{dx}{dt} = f_1(x, y, t)$$

$$\frac{dy}{dt} = f_2(x, y, t)$$

If $f_1$ and $f_2$ are linear in both variables $x$ and $y$ then the system is called **linear**, otherwise the system is called **nonlinear**.

Note that the theory of $n$ linear first-order systems of DEs is inextricably linked to the theory of linear $n$-th order DEs. For example, if $y'' = f(t, y(t)), y'(t))$ and one makes the substitution $u_2 = y'(t)$ and $u_1 = y(t)$ this second-order DE can be converted into a system of 2 first-order DEs.

EXAMPLE

Let's write $y'' = f(t, y(t)), y'(t))$ as a system of first-order DEs.

**Exercise**

Show that the third-order equation $y^{(3)} + 3y'' + 2y' - 5y = \sin(2t)$ can be written as a linear system of first-order DEs.

## 2. Lotka-Volterra Model

Probably the most famous system of ordinary differential equations of all time is the **Lotka-Volterra predator-prey model**. (The S-I-R model of epidemics would be a close second.) $R(t)$ denotes the population of prey (rabbits) and $F(t)$ denotes the population of their predators (foxes). Let $a$, $b$, $c$ and $d$ be positive parameters; the model is represented by:

$$\frac{dR}{dt} = aR - bRF \quad (1)$$

$$\frac{dF}{dt} = cRF - dF \quad (2)$$

$$R(0) = R_0 \quad F(0) = F_0$$

EXAMPLE

Let's interpret the Lotka-Volterra predator-prey model for various parameter "regimes."

**1.** What happens to the populations of rabbits and foxes over time if $b = c = d = 0$ (remember $a > 0$)?

**2.** What happens to the populations of rabbits and foxes over time if $a = b = c = 0$ (remember $d > 0$)?

**3.** What is the significance of the $RF$ terms in the model? Is the significance the same for the rabbits and for the foxes?

**4.** What does the model predict will happen if at any time one of the populations of the rabbits or the foxes becomes zero?

GroupWork

Are there any fixed points for the system of equations? (In other words, find the stationary points or equilibria of the system of DEs.)

## 3. Exploring the Predator-Prey model

$$\frac{dR}{dt} = 2R - 1.2RF$$
$$\frac{dF}{dt} = -F + 0.9RF$$

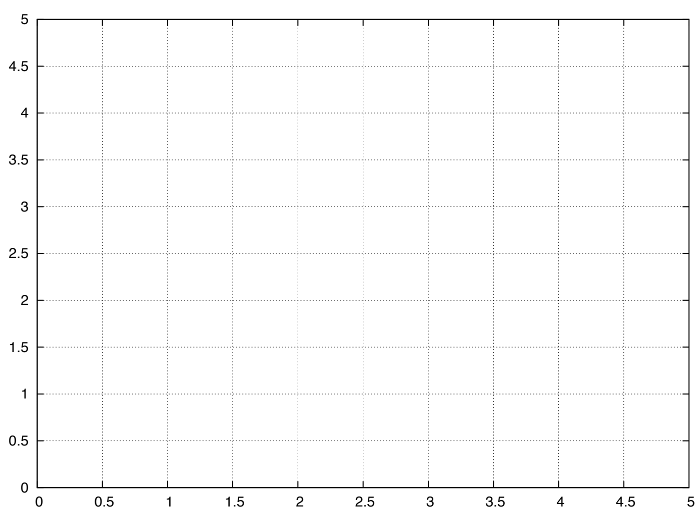
EXAMPLE

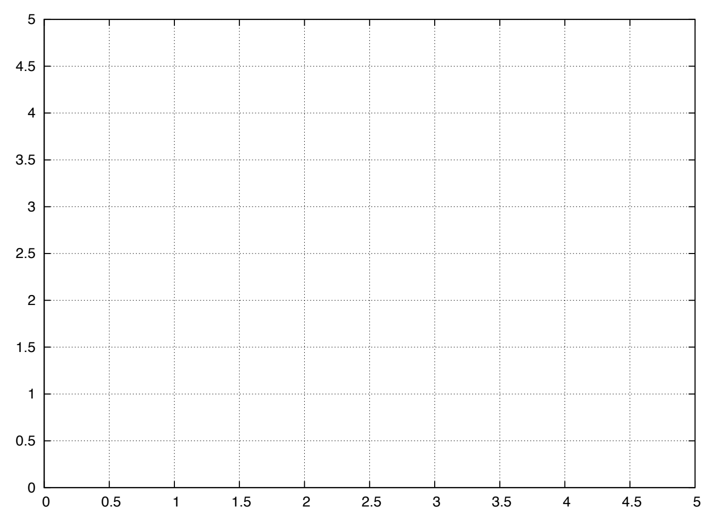
Using the program `HPGSystemSolver` (available on the S-drive at `S:\Math Courses\Math341\Buckmire\DifferentialEquations.exe` or `BDH.jar`), let's explore the solution curves for different initial conditions if a=2, b=1.2 c=0.9 and d=1.

Describe what you see when you use the intial condition $R(0) = 1, F(0) = 1$? What happens as you approach the initial guess $R(0) = 1.1, F(0) = 1.67$?

(NOTE: The HPGSystemSolver plot solutions to $x' = f(x, y), \quad y' = f(x, y)$ so you'll have to transform the Lotka-Volterra Equation into this form.)

Sketch pictures of what you see in the space below. On the left, sketch the phase portrait, and on the right sketch solution curves of $R(t)$ and $F(t)$ versus $t$.




What can you say are the long-term implications of the Lotka-Volterra Model starting at $R(0) = F(0) = 1$? What happens to $R$ and $F$ as $t \to \infty$?

## 4. The Modified Predator-Prey model

$$\frac{dR}{dt} = 2R\left(1 - \frac{R}{2}\right) - 1.2RF$$
$$\frac{dF}{dt} = -F + 0.9RF$$

**1.** What is the significance of the new terms in the modified predator-prey model that did not appear in the Lotka-Volterra model?

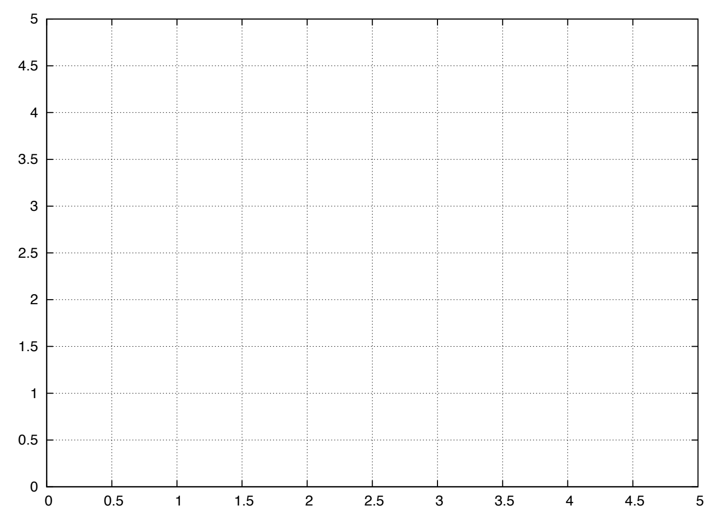
**2.** What happens to the populations of rabbits if the number of foxes becomes zero?

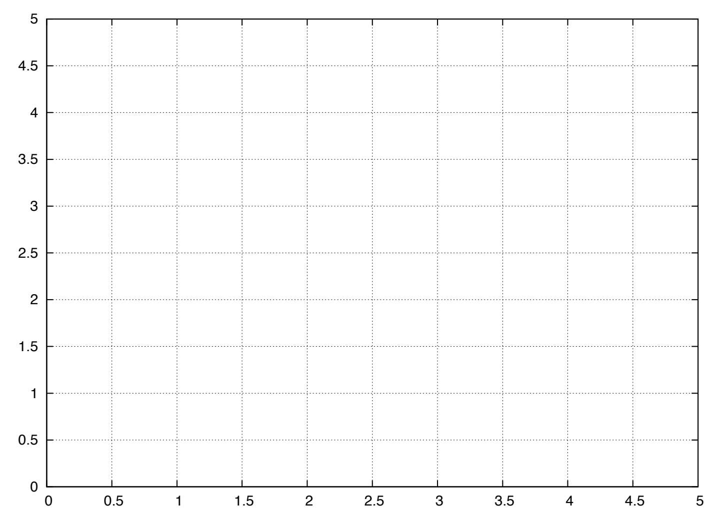
**3.** What happens to the populations of foxes if the number of rabbits becomes zero?

**4.** Are there any equilibrium points of this system of DEs?

Describe what you see when you use the intial condition $R(0) = 1, F(0) = 1$? What happens as you approach the initial guess $R(0) = 1.1, F(0) = 1.67$?

Sketch pictures of what you see in the space below. On the left, sketch the phase portrait, and on the right sketch solution curves of $R(t)$ and $F(t)$ versus $t$.




What can you say are the long-term implications of the modified Predator-Prey Model starting at $R(0) = F(0) = 1$? What happens to $R$ and $F$ as $t \to \infty$?