Differential Equations

Math 340 §2 Fall 2015 — MWF 3:00-3:55pm Fowler 307

 — http://sites.oxy.edu/ron/math/340/15/

Worksheet 4

TITLE Euler's Method
CURRENT READING Blanchard, 1.4

Homework Set #3 due Friday September 11
Section 1.4: 2, 6, 11, 15. **Section 1.5**: 2, 3, 12, 14, 15.
Section 1.6: 2, 7, 8, 19, 20, 30, 31, 41.

SUMMARY
We will learn about a simple (and possibly familiar) numerical technique called Euler's Method which approximates solutions to ODEs quantitatively.

Euler's Method

Given an expression for how the derivative of an unknown function $y(x)$ changes, i.e. $y' = f(x, y)$, and an initial value $y(x_0) = y_0$ one can use Euler's Method to estimate $y(x)$ at a point close by with bounded error.

$$y(x_{new}) = y(x_{old}) + \Delta y \text{ where } \Delta y \approx y'(x_{old})\Delta x$$

In other words
$y_{new} \approx y_{old} + y'_{old}\Delta x$ and $x_{new} = x_{old} + \Delta x$ or $y_{k+1} \approx y_k + f(x_k, y_k)\Delta x$ and $x_{k+1} = x_k + \Delta x$.

Exercise

Draw a picture of the Euler approximation y_{new} starting at a point (x_{old}, y_{old}). Is the slope field involved?

EXAMPLE

Using Euler's Method To Approximate Solutions To Differential Equations

1. Suppose y changes with time t according to the equation $y' = 1 + 2y$.
(a) What is the rate of change of y when $y = 3$?
(b) Suppose when $t = 0$, $y = 3$. Use Euler's Method with $\Delta t = .5$ to estimate $y(1)$.
(c) Is your estimate of $y(1)$ an over-estimate or under-estimate?

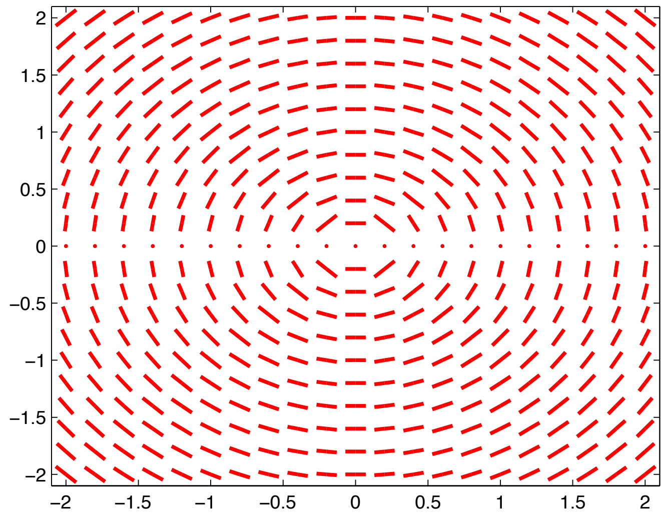
To use Euler's Method generally the following table can be helpful

x	y	y'	Δy

Slope Fields and Euler's Method

Exercise

Consider the differential equation $y' = -x/y$ with initial condition $y(0) = 1$. Given that the exact solution is $y(x) = \sqrt{1 - x^2}$,
(a) use the slope field to estimate $y(1/2)$ for the solution that satisfies the given initial condition.
(b) Compare your estimate with the exact value of $y(1/2)$
(c) Use Euler's Method with $\Delta x = .25$ to estimate $y(1/2)$.
(d) Is your Euler's Method estimate and over-estimate or under-estimate? Explain why.



To use Euler's Method generally the following table can be helpful

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Numerical Error in Using Euler's Method

GroupWork

Complete the following sentences:
As the time step Δt ________________ in magnitude, the numerical error in computing $y(x_1)$ using Euler's Method decreases in magnitude.
As the time step Δt ________________ in magnitude, the numerical error in computing $y(x_1)$ using Euler's Method increases in magnitude.
When y'' is ____________________________________ on $x_0 < x < x_1$ the function $y(x)$ is concave up and estimates of $y(x_1)$ using Euler's Method will be ______________________________.
When y'' is ____________________________________ on $x_0 < x < x_1$ the function $y(x)$ is concave down and estimates of $y(x_1)$ using Euler's Method will be _____________________________.