Differential Equations

Math 340 §2 Fall 2015 MWF 3:00-3:55pm Fowler 307

 http://sites.oxy.edu/ron/math/340/15/

Class 2: **Friday August 28**

TITLE Separation of Variables
CURRENT READING Blanchard, §1.2 and §1.3
HOMEWORK (Due Friday September 4)
Section 1.1: 2, 3, 13,14. Section 1.2: 1, 2, 3, 6, 21, 27, 32. Section 1.3: 7, 8, 12, 13,14,16.

SUMMARY
In today's class we shall review an analytical technique for solving a particular class of first-order (separable) ODEs known as **Separation of Variables**.

1. Solving Separable Differential Equations

DEFINITION: **separable DE**

A **separable first-order differential equation** is one which has the form $\dfrac{dy}{dx} = g(x)h(y)$
The technique for solution is to separate the variables in the equation by placing everything with an independent variable on one side, and everything with a dependent variable on the other. This produces:

$$\frac{dy}{h(y)} = g(x)dx$$

One can then treat each side of the equation as an indefinite integral,

$$\int \frac{dy}{h(y)} = \int g(x)dx$$

which, if each function $1/h(y)$ and $g(x)$ have anti-derivatives $H(y)$ and $G(x)$, respectively produces

$$H(y) = G(x) + C$$

The above equation thus defines (implicitly) a **1-parameter family of solutions** to the given first-order DE. When an initial condition $y(a) = b$ is also given, then a **particular solution** can be obtained.

EXAMPLE
Let's consider the Malthusian Model of population $P' = kP, P(0) = P_0$ and obtain the solution to this **initial value problem** by separation of variables.

NOTE: k is a parameter in the Malthusian population model, which has P as a **dependent** variable and t as an **independent variable**.

Exercise

Let's consider the Verhulst or Logistic Model of Population $P' = kP(1-P/N)$, $P(0) = P_0$. What are the interpretation of the parameters k and N in the Verhulst model?

GROUPWORK

Show that if you make the change of variables $Q(t) = P(t)/N$ the Logistic Model can be written as $Q' = kQ(1-Q)$ which has a solution of the form $Q(t) = \dfrac{1}{1+Ce^{-kt}}$ where C is any real number.

The particular solution to the given IVP for the Verhulst model is $P(t) = \dfrac{P_0 N}{P_0 + (N - P_0)e^{-kt}}$