Topic: Laplace Transforms

The learning goal of this quiz is to provide you with an opportunity to demonstrate your comfort with integral operators and to introduced you to an important special function.

Reality Check:

EXPECTED SCORE: __________/10  ACTUAL SCORE: __________/10

Instructions:

0. Please look for a hint on this quiz posted to http://sites.oxy.edu/ron/math/340/15/

1. Once you open the quiz, you have 30 minutes to complete it, please record your start time and end time at the top of this sheet.

2. You may use the book or any of your class notes. You must work alone.

3. If you use your own paper, please staple it to the quiz before coming to class. If you don’t have a stapler, buy one. QUZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.

4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.

5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution. Use complete sentences wherever possible.

6. Relax and enjoy...

7. This quiz is due at the beginning of class on Monday November 16, in class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, _______________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.
1. The **Gamma Function** $\Gamma(\alpha)$ is defined below as

$$\Gamma(\alpha) = \int_0^\infty e^{-t}t^{\alpha-1} \, dt, \quad (\alpha > 0).$$

(a) 1 point. Show that $\Gamma(1) = 1$.

(b) 2 points. Show that $\Gamma(\alpha + 1) = \alpha \Gamma(\alpha)$.

(c) 3 points. Use the results given in (a) and (b) to show that $\Gamma(n + 1) = n!$, where $n$ is a positive integer. (HINT: use mathematical induction).

(d) 4 points. Use the previous results ((a), (b) and (c)) to compute the Laplace transform of $t^\alpha$ (for $\alpha > 0$). In other words, show that $\mathcal{L}[t^\alpha] = \frac{\Gamma(\alpha + 1)}{s^{\alpha+1}}$ and $\mathcal{L}[t^n] = \frac{n!}{s^{n+1}}$ when $n$ is a positive integer.